DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA 93943-8002

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

P14571

A COMPUTER CODE (USPOTF2)
FOR UNSTEADY INCOMPRESSIBLE FLOW
PAST TWO AIRFOILS

by

Chung-Khiang Pang

September 1988

Thesis Advisor M.F. Platzer

Approved for public release; distribution is unlimited.

T242226

nclassified

ecurity classification of this page

REPORT DOCUMENTATION PAGE

1a Report Security Classification Unclassified	1b Restrictive Markings
2a Security Classification Authority	3 Distribution Availability of Report Approved for public release; distribution is unlimited.
2b Declassification Downgrading Schedule	
4 Performing Organization Report Number(s)	5 Monitoring Organization Report Number(s)

6a Name of Performing Organization Naval Postgraduate School	6b Office Symbol *(if applicable)* 67	7a Name of Monitoring Organization Naval Postgraduate School
6c Address *(city, state, and ZIP code)* Monterey, CA 93943-5000		7b Address *(city, state, and ZIP code)* Monterey, CA 93943-5000
8a Name of Funding Sponsoring Organization	8b Office Symbol *(if applicable)*	9 Procurement Instrument Identification Number
8c Address *(city, state, and ZIP code)*		10 Source of Funding Numbers

Program Element No	Project No	Task No	Work Unit Accession No

11 Title *(include security classification)* A COMPUTER CODE (USPOTF2) FOR UNSTEADY INCOMPRESSIBLE FLOW PAST TWO AIRFOILS

12 Personal Author(s) Chung-Khiang Pang

13a Type of Report	13b Time Covered	14 Date of Report *(year, month, day)*	15 Page Count
Master's Thesis	From To	September 1988	169

16 Supplementary Notation The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17 Cosati Codes

Field	Group	Subgroup

18 Subject Terms *(continue on reverse if necessary and identify by block number)* Computational Fluid Dynamics, Panel Methods, Unsteady Aerodynamics, Potential Flows.

19 Abstract *(continue on reverse if necessary and identify by block number)*

A numerical code, USPOTF2, has been formulated to solve for the potential flow for two airfoils executing unsteady motions in an inviscid incompressible flow medium. This code is an extension of an existing code U2DIIF, which does the same calculations for the single airfoil case. The technique uses the well known Panel Methods for steady flow and extends it to unsteady flow by introducing a wake model which creates a non-linear problem due to the continuous shedding of vortices into the trailing wake. The presence of the second airfoil introduces a set of non-linear coupled equations for the Kutta condition. Numerous case-runs are presented to illustrate the capability of the code. The case of the step change in angle of attack is compared with Giesing's work. All other case-runs are illustrated together with the results for the single airfoil case.

20 Distribution Availability of Abstract ☒ unclassified unlimited ☐ same as report ☐ DTIC users	21 Abstract Security Classification Unclassified
22a Name of Responsible Individual M.F. Platzer	22b Telephone *(include Area code)* (408) 646-2311
	22c Office Symbol 67Pl

DD FORM 1473,84 MAR

83 APR edition may be used until exhausted
All other editions are obsolete

security classification of this page

Unclassified

Approved for public release; distribution is unlimited.

A Computer Code (USPOTF2)
for Unsteady Incompressible Flow
past Two Airfoils

by

Chung-Khiang Pang
Civilian, Singapore Ministry of Defence
B.Eng.(Hons), University of Adelaide, South Australia, 1981

Submitted in partial fulfillment of the
requirements for the degrees of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING
and
AERONAUTICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL
September 1988

ABSTRACT

A numerical code, USPOTF2, has been formulated to solve for the potential flow for two airfoils executing unsteady motions in an inviscid incompressible flow medium. This code is an extension of an existing code U2DIIF, which does the same calculations for the single airfoil case. The technique uses the well known Panel Methods for steady flow and extends it to unsteady flow by introducing a wake model which creates a non-linear problem due to the continuous shedding of vortices into the trailing wake. The presence of the second airfoil introduces a set of non-linear coupled equations for the Kutta condition. Numerous case-runs are presented to illustrate the capability of the code. The case of the step change in angle of attack is compared with Giesing's work. All other case-runs are illustrated together with the results for the single airfoil case.

Thesis
H1571
C.1

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available. to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

TABLE OF CONTENTS

I. INTRODUCTION 1
  A. GENERAL 1
  B. BASIC THEORY AND APPROACH 1
    1. Steady Flow Problem 1
    2. Unsteady Flow Problem 2
  C. SCOPE 3

II. STEADY FLOW PROBLEM FORMULATION FOR TWO AIRFOILS 5
  A. GENERAL 5
  B. FRAMES OF REFERENCE 5
  C. STEADY FLOW PANEL METHODS 5
    1. Definition of nodes and panels 5
    2. Distribution of Singularities 7
    3. Boundary Condition 10
  D. INFLUENCE COEFFICIENT 10
    1. Concept of Influence Coefficient 10
    2. Notation for Influence Coefficient 11
    3. Computation of Influence Coefficient 11
  E. NUMERICAL SOLUTION SCHEME 14
    1. Rewriting the Boundary Condition 14
    2. Solving for the Strengths of Source and Vorticity Distribution 15
    3. Computation of Velocity and Pressure Distribution 15
    4. Computation of Forces and Moments 16

III. UNSTEADY FLOW PROBLEM FORMULATION 18
  A. GENERAL 18
  B. FRAMES OF REFERENCE 18
  C. UNSTEADY FLOW MODEL 19
    1. Rigid Body Motion 19
    2. Wake Model 21
    3. Additional Boundary Conditions 21

D. INFLUENCE COEFFICIENTS 23
    1. More A's and B's Influence Coefficients 23
    2. New C's Influence Coefficient 24
    3. Computation of the Time-Dependent Influence Coefficient 25
E. NUMERICAL SOLUTION SCHEME 26
    1. The Flow Tangency Condition 26
    2. The Kutta Condition 28
    3. Convection of the Wake 32
    4. Disturbance Potential and Pressure Distribution 32
    5. Computation of Forces and Moments 36

IV. DESCRIPTION OF COMPUTER CODE USPOTF2 37
A. PROGRAM STRUCTURES AND CAPABILITIES 37
    1. Restrictions and Limitations 37
    2. Current Structures of USPOTF2 Main Program 37
B. DESCRIPTION OF SUBROUTINES 43
    1. Subroutine BODY 43
    2. Subroutine COEF 43
    3. Subroutine COFISH (deleted for the two airfoil case) 43
    4. Subroutine CORVOR 43
    5. Subroutine FANDM 43
    6. Subroutine GAUSS 44
    7. Subroutine INDATA 44
    8. Subroutine INFL 44
    9. Subroutine KUTTA 45
    10. Subroutine NACA45 45
    11. Subroutine NEWPOS 45
    12. Subroutine PRESS 46
    13. Subroutine SETUP 46
    14. Subroutine TEWAK 46
    15. Subroutine VELDIS 46
C. INPUT DATA FOR PROGRAM USPOTF2 47
D. OUTPUT DATA FROM PROGRAM USPOTF2 47

V. RESULTS AND DISCUSSION OF CASE-RUNS 53
A. STEP CHANGE IN ANGLE OF ATTACK 53
1. Case Run Definition 53
2. Differences between USPOTF2 and Giesing's code 53
3. Results and Discussions 54
B. OTHER SUB-CASES 70
1. Modified Ramp Change in Angle of Attack 71
2. Translational Harmonic Motion 71
3. Rotational Harmonic Motion 71

VI. CONCLUSION 90
A. GENERAL COMMENTS 90
B. ENHANCING USPOTF2 PROGRAM CAPABILITY 90

APPENDIX A. USPOTF2 SOURCE LISTINGS 92

APPENDIX B. EXAMPLE INPUT DATA FOR PROGRAM USPOTF2 135

APPENDIX C. EXAMPLE OUTPUT DATA FROM PROGRAM USPOTF2 136

LIST OF REFERENCES 154

INITIAL DISTRIBUTION LIST 155

LIST OF FIGURES

Figure 1. Frames of Reference for Steady Flow. 6
Figure 2. Panel Methods Representation for Steady Flow. 8
Figure 3. Influence Coefficient for Source Panel 12
Figure 4. Frames of Reference for Airfoil 1 20
Figure 5. Panel Methods Representation for Unsteady Flow 22
Figure 6. Flow Chart for USPOTF2 Computer Code. 40
Figure 7. List of Input Variables. 48
Figure 8. List of Output Variables. 51
Figure 9. Giesing's Calculated Results 56
Figure 10. USPOTF2 Results obtained with USPOTF2 for equal velocities at the trailing edge. 58
Figure 11. USPOTF2 Results obtained with USPOTF2 for equal pressures at the trailing edge. 61
Figure 12. Pressure Distributions with different Kutta condition. 64
Figure 13. Pressure Distribution and Lift History as a function of the Vertical distance between the airfoils. 65
Figure 14. Step Change in Angle Of Attack 67
Figure 15. Modified Ramp Change in Angle Of Attack 73
Figure 16. Harmonic Plunging Motion 79
Figure 17. Harmonic Pitching Motion 84

TABLE OF SYMBOLS

A	singularity-type indicator for uniformly distributed source
B	singularity-type indicator for uniformly distributed vorticity
C	singularity-type indicator for concentrated point vortex
c	chord length
C_d	2-dimensional drag coefficient
C_l	2-dimensional lift coefficient
C_{l_∞}	steady state value of C_l
C_m	2-dimensional pitching moment coefficient about leading edge
C_{m_∞}	steady state value of C_m
C_p	pressure coefficient
C_x	x-force coefficient of local system (in the code)
C_y	y-force coefficient of local system (in the code)
h_x	chordwise translational position (positive forward)
h_y	transverse translational position (positive downward)
M	Mach number
m	number of core vortices
n	total number of panels
i, j	unit vectors along the x and y directions of respective frame of reference
n, t	unit vectors normal and tangential to panel
P	static pressure
P_∞	freestream static pressure
q	unit strength of uniformly distributed panel source
SS	perimeter length of airfoil
t	time step
V_∞	freestream velocity vector
V^n	total velocity component normal to panel
V^t	total velocity component tangential to panel
U, V	absolute velocity components in the x and y directions
x-y	coordinate system fixed on the airfoil

X-Y	coordinate system fixed with respect to the free stream
xm,ym	mid point of panel
α, **AOA**	angle of attack (positive clockwise from V_∞)
α_i	initial angle of attack
Γ	dimensionless circulation strength
γ	unit strength of uniformly distributed panel vorticity
Δ	length of the shed vorticity panel
$\delta\alpha$	change in AOA from α_i or amplitude of pitch oscillation
δh_x , δh_y	amplitude of chordwise and transverse oscillations
Θ	orientation angle of the shed vorticity panel with x-direction
θ	inclination angle of the panel with the x-direction
λ	phase difference of the chordwise from the transverse oscillation
ρ	incompressible density
τ	dimensionless rise time for ramp change in AOA
Φ	total velocity potential
ϕ_∞	velocity potential due to free-stream
ϕ	disturbance velocity potential
ϕ_s	velocity potential due to source distribution
ϕ_v	velocity potential due to vortex distribution
ϕ_{cv}	velocity potential due to core vortices in the wake
Ω	pitch angular velocity of airfoil (positive counterclockwise)
ω	harmonic oscillation frequency
κ	iteration counter

ACKNOWLEDGEMENTS

The research for this thesis was conducted at the Naval Postgraduate School Computer Center facilities. Work was performed under the guidance of Professor Platzer, thesis advisor. I would like to express my sincere appreciation to him for providing me with the opportunity to work in the area of computational fluid dynamics and for his patience and encouragement, both in the course of the research as well as the final preparation of the manuscript. I would also like to thank Professor Howard, second reader, who patiently reviewed the manuscript and gave encouragement at a critical stage.

My sincere appreciation also goes to Mr. Andreas Krainer, whose total familiarity and understanding of the original code made a critical difference in the coding of USPOTF2; especially encouraging was the number of occasions, too numerous to recall, where he would give expert advice as to the possible coding areas which might be in error during troubleshooting days and finally his constructive and thorough review in going through every single formula in the manuscript.

Finally, I wish to thank my very loving wife and co-typist, Ling Yu, who bore the tremendous extra demands of the final months with patience, understanding and encouragement and who toiled painstakingly with successive drafts, cheerfully accepting my determination to alter every section after, but never before, it had been typed.

I. INTRODUCTION

A. GENERAL

In this paper , a numerical method is formulated to solve for the flow about two two-dimensional airfoils which are arbitrary located and are performing an arbitrary time dependent motion in an inviscid incompressible fluid. The original work by Teng [Ref. 1] is for a single arbitrarily defined airfoil in the same potential flow condition. The extension of Teng's code to two bodies is considered here. Where possible, the same convention and notation are adopted. As for the single airfoil case, all velocities are non-dimensionalised with respect to the free stream and all lengths with respect to the chord length.

A new subroutine (SUBROUTINE NEWPOS) is added to transform either of the two local coordinate systems to the global coordinate system. A modification is entered into the treatment of the Kutta condition to make it consistent with the unsteady flow Kutta condition treatment. Subroutine COFISH is deleted and its role is taken over by Subroutine COEF which normally performs the formulation for the flow tangency condition for the unsteady case. A more accurate method is also introduced to obtain the velocity potential. The reader is referred to the work of Krainer [Ref. 2] for further improvement of the original code.

This documentation is set up as for the original documentation in order that it will be easier to follow and cross-referenced. While it is the intent of the author to keep this thesis as complete as possible, the reader is well advised to review Reference 1 for the work involved in the single airfoil case; no special effort will be made to reproduce it here.

B. BASIC THEORY AND APPROACH

1. Steady Flow Problem

The treatment for the two airfoils case in steady incompressible flow follows closely to the single airfoil case. The governing equation, as for the single airfoil case , follows from the Conservation of Mass and the Condition of Irrotational Flow.

The continuity equation of an incompressible fluid ($div\ \vec{V} = 0$) and the condition of irrotational flow ($curl\ \vec{V} = 0$) leads to the well known Laplace Equation.

$$\mathrm{div}(\,\mathrm{grad}\phi\,) = 0 \tag{1.1}$$

with ϕ denoting the disturbance potential for the velocity. This is seen to be the classic Neumann problem of potential theory with the usual problem of defining the boundary conditions. The boundary conditions for the disturbance potential ϕ are that its gradient normal to the surface be equal to the normal velocity of the surface and that its gradient vanish at infinity; that is

$$\nabla\phi \bullet \vec{n} = \vec{V}_r \bullet \vec{n} \; on \; S \tag{1.2}$$

$$\lim_{P \to \infty} \nabla\phi(P) \to 0 \tag{1.3}$$

where $\vec{V}_r$ is the resultant velocity of a point on the body as seen from the inertia frame of reference, $\vec{n}$ is the outward unit vector normal to the body, S is the body surface and P represents a general point. Equation 1.2 holds for both airfoils i.e. on both surfaces.

The pressure coefficient is obtained through the Bernoulli's equation which derives from the Momentum equation.

The approach adopted is associated with Hess and Smith [Ref. 3] who devised the popularly known *PANEL* method in the early sixties. In words, the boundary or airfoil surfaces S_1 and S_2 about which the flow is to be computed is approximated by a large number of surface elements whose characteristic dimensions are small compared to those of the body. Over each surface element, a uniform source distribution and a uniform vorticity distribution is placed. The source strength (q_j) varies from element to element, while the vortex strength (γ_l) is the same for all elements in the same airfoil but is different across the airfoil. The singularity strengths are determined from the flow tangency condition on both body surfaces and the two Kutta conditions at both trailing edges. With the determination of the singularity strengths, the relevant aerodynamic data can then be subsequently computed.

2. Unsteady Flow Problem

The unsteady problem is similar to the steady flow problem in that they both have the same governing equation viz. the Laplace equation, and that for both problems, the pressure and velocity are decoupled so that the velocity and pressure calculation can be computed separately and consecutively.

This problem differs from the steady flow in that another model is required to simulate the continuous shedding of vorticity into the trailing wake. The existence of a vortex sheet behind the airfoil can be explained by the Helmholtz theorem which is basically a statement of the Conservation of Vorticity. This requires that any change in the circulation around the airfoil must be matched by an equal and opposite vortex somewhere in the flowfield. The presence of the countervortices provides the flow with a kind of a memory in that the flow at a particular time is affected by the bound circulation of the past. It is this non-linearity that distinguishes the numerical technique from the simple steady flow problem of solving N linear equations in N unknowns.

The solution technique requires an iterative type solution. The present approach follows closely the original panel method of Hess and Smith as described in the steady flow development, while with regard to the modelling of the wake, it adopts the procedure advocated by Basu and Hancock [Ref. 4]. A uniform source distribution $(q_j)_k$ and a uniform vorticity distribution $[\gamma(l)]_k$ as for steady flow is placed on each panel at time t_k where j denotes the panel number and l the airfoil number. The wake consists of a single vorticity panel attached as an additional element on each airfoil through which the vorticities are shed into the respective wake and a series of point vortices which are being convected downstream with the fluid. A uniform vorticity distribution of strength $(\gamma_w(l))_k$ is placed on the wake panel of each airfoil. This panel is further characterised by its length $\Delta(l)_k$ and its inclination $(\Theta_{(n-1)l})_k$ with respect to the respective local frame of reference. After each time step, the vorticity of the wake panel is concentrated into a single point vortex and convected downstream. Simultaneously a new wake panel is formed. The downstream wake of point vortices is thus formed by the shed vorticity of previous time steps.

C. SCOPE

Chapter II extends the original code to handle the steady flow problem for two airfoils set at different relative distances and angles of attack.

Chapter III deals with the unsteady problem for the two airfoils system. It introduces a new subroutine and a more accurate method of calculating the velocity potential. The Kutta condition for the two airfoils system is specially treated as its unique problem of a non-linear coupled system is not seen in the single airfoil case.

Chapter IV describes the computer program, its essential capabilities and limitations, its associated subroutines, its input requirements and its associated output printout.

Chapter V presents the results of some case-runs. Of interest is a comparison case of a step change in angle of attack (AOA) with Giesing [Ref. 5] for the same airfoil undergoing an impulsive start at the same AOA. Case-runs will also be run with both airfoils at large distances apart to compare with the single airfoil case. In addition, example cases for which no comparisons exist are given, to exhibit the capability of the method.

Finally, Chapter VI concludes with future development efforts and the application potential of this numerical method.

II. STEADY FLOW PROBLEM FORMULATION FOR TWO AIRFOILS

A. GENERAL

The modification work for the steady flow is straightforward. The revised program allows for two arbitrarily defined airfoils placed at an arbitrary distance set at different angles of attack. For reason of simplicity, the number of panels and nodes and the pivot location are set to be the same for both airfoils.

B. FRAMES OF REFERENCE

Three frames of reference are involved in the two two-dimensional airfoils' case in steady flow. These are three inertia frames of references as indicated in Figure 1.

The first inertia frame of reference (also known as global frame of reference) is set at the pivot position of the first airfoil with the X-axis pointing in the direction of the free-stream velocity. The two other inertia frames of references, henceforth, will be known as two frozen local frames of reference[1] (x_1y_1 and x_2y_2) are fixed respectively to each airfoil with the x-axis coinciding with the chord line originating from the respective leading edge. The two local frames of reference are set apart by XShift and YShift on the global frame of reference. In steady flow, the fluid velocity and pressure depend only on the spatial coordinates (X,Y) and not on time.

The two airfoils are defined in the local coordinate system as input data for simplicity and are then transformed to the global coordinate system through a knowledge of the relative positions of the 2 airfoils' pivots positions and the respective local angles of attack.

C. STEADY FLOW PANEL METHODS

1. Definition of nodes and panels

Each airfoil surface is divided into n straight line segments called panels by (n+1) arbitrarily chosen points called nodes. The numbering sequence begins with panel 1 on the lower surface at the first airfoil trailing edge and proceeds clockwise around the airfoil contour so that the last panel on airfoil 1 ends on the upper surface at the trailing edge. This numbering sequence then proceeds in a similar fashion for the second airfoil (See Figure 2). As with the single airfoil case, the numbering sequence

---

[1] For the steady case, the notation 'frozen' will be dropped

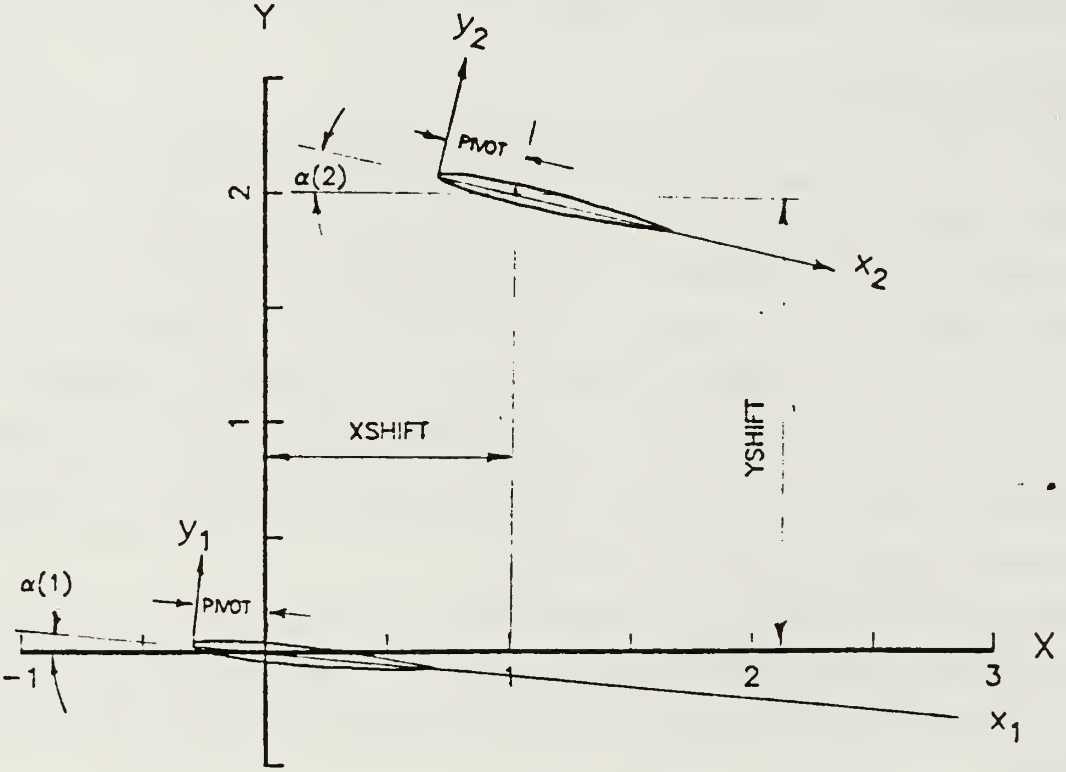


Figure 1. Frames of Reference for Steady Flow.

dictates that the airfoil body always lies on the right hand side of the i^{th} panel as one proceeds from the i^{th} node to the $(i+1)^{th}$ Also the 1^{st} and $(n+1)^{th}$ nodes coincide at the first airfoil trailing edge and the $(n+2)^{th}$ and $(2n+2)^{th}$ coincide at the second airfoil trailing edge. This numbering system facilitates the definitions of the unit normal vector n_i and the unit tangential vector t_i for all panels with n_i being directed outward from the body into the flow and t_i directed from the i^{th} node to the $(i+1)^{th}$ node. The numbering of the panel system is somewhat complicated by the fact that a continuous panel numbering sequence across the airfoil is desired. This procedure leads to the peculiar panel numbering behaviour seen in Figure 2 where for the first airfoil, the i^{th} panel lies between i^{th} and $(i+1)^{th}$ nodes while for the second airfoil, the i^{th} panel lies between $(i+1)^{th}$ and $(i+2)^{th}$ nodes[2].

2. Distribution of Singularities

Over each surface element of the two airfoils, a uniform source distribition and a uniform vorticity distribution is placed. The source strength q_j varies from element to element for each airfoil while the vorticity strength γ_l remains the same for all elements in the same airfoil but is different across the airfoil. This choice of singularities follows closely the original panel method of Hess and Smith. It automatically satisfies the Laplace Equation (which is the governing equation for the inviscid incompressible flow) and the boundary condition at the far field (∞). In addition, as the Laplace Equation is a linear homogeneous second order partial differential equation, an overall complicated flow field can be built up by the combination of simple flows with the condition that the appropriate boundary condition on the airfoil be satisfied accurately.

For our case, the overall flow field (represented by the velocity potential Φ) can be built up by three simple flows. Writing this in terms of the respective local frame of reference,

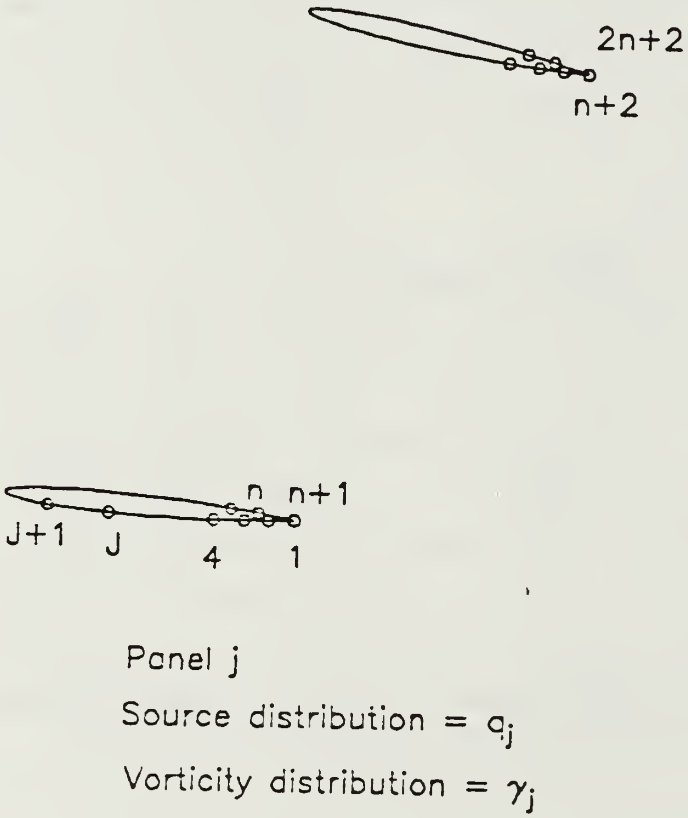
$$\Phi(x\,,y) = \phi_\infty(x\,,y) + \phi_s(x\,,y) + \phi_v(x\,,y) \tag{2.1}$$

where $\phi_\infty(x\,,y)$ is the potential of the onset flow,

$$\phi_\infty(x\,,y) = V_\infty[x\mathrm{Cos}\,\alpha(l) + y\mathrm{Sin}\,\alpha(l)] \tag{2.2}$$

---

[2] For the code θ_i has been defined out of the above convention with θ_i, i = 1,2...n as per panel numbers for the first airfoil, but with θ_{n-1} reserved for the wake element of the first airfoil and θ_i, i = n + 2,n + 3...2n + 1 for the second airfoil with θ_{2n-2} reserved for the wake element of the second airfoil for unsteady flow.


2n+2
n+2
n
n+1
J+1
J
4
1
Panel j
Source distribution = q_j
Vorticity distribution = γ_j

Note: 1. Nodal points defined by i = 1,2 ... n+1 for first airfoil
and i = n+2 ... 2n+2 for second airfoil.

2. Panel number defined by j = 1,2 ... n for first airfoil
and j = n+1 ... 2n for second airfoil.

Figure 2. Panel Methods Representation for Steady Flow.

ϕ_s is the velocity potential of a source of distribution q(s) per unit length.

$$\phi_s = \int \frac{q(s)}{2\pi} \ln r \, ds \tag{2.3}$$

ϕ_v is the velocity potential of a vorticity distribution γ (s) per unit length

$$\phi_v = -\int \frac{\gamma(s)}{2\pi} \theta \, ds \tag{2.4}$$

At this point, the disturbance potential, ϕ, is introduced, which is defined to be the sum of the potential due to the source and vorticity distribution.

$$\phi = \phi_s + \phi_v \tag{2.5}$$

Equation (2.1) can then be read as

$$\Phi = \phi_\infty + \phi \tag{2.6}$$

The convenience of defining the above allows for the total velocity vector to be viewed as two components viz. the onset and the induced velocity due to the disturbance potential. The total velocity is thus :

$$\begin{aligned} \vec{V}_{total} &= \nabla\Phi \\ &= \nabla\phi_\infty + \nabla\phi \\ &= \vec{V}_\infty + \nabla\phi \end{aligned} \tag{2.7}$$

It is in the introduction of the disturbance potential that leads us to the concept of influence coefficient which will be elaborated in a later section.

The pressure coefficient can be obtained from Bernoulli's Equation which is derived from the Conservation of Momentum.

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho V_\infty^2} = 1 - \left\{ \frac{V_{total}}{V_\infty} \right\}^2 \tag{2.8}$$

3. Boundary Condition

As in the single airfoil case, the boundary conditions to be satisfied include the flow tangency conditions and the Kutta condition. The flow tangency conditions are satisfied at the exterior mid-points (control points). The normal velocity is taken with respect to the respective local frame of reference (for consistency with unsteady flow notation to be introduced later):

$$(V^n)_i = 0, \qquad i = 1,2 \ldots n, n+1 \ldots 2n \tag{2.9}$$

where $(V^n)_i$ is the normal component of the total velocity[3].

The Kutta condition postulates that the pressure on the upper and lower surface at the trailing edge of each airfoil be equal. For steady potential flow, equal pressure implies equal tangential velocity in the downstream direction at the first and last panel of each airfoil viz the Bernoulli equation. With our definition of the tangential vector we then have

$$(V^t)_1 = -(V^t)_n \qquad \mathit{1st\ airfoil}$$

$$(V^t)_{n+1} = -(V^t)_{2n} \qquad \mathit{2nd\ airfoil} \tag{2.10}$$

As with the single airfoil, equations 2.9, and 2.10 lead to a linear system of (2n+2) simultaneous equations. With V^n and V^t expressed explicitly in terms of q_j (j = 1, 2, ... n. n+1... 2n) and γ_l (l = 1, 2) we have 2n+2 unknowns in (2n+2) system of linear simultaneous equations which can be easily solved.

D. INFLUENCE COEFFICIENT

1. Concept of Influence Coefficient

As introduced earlier, this important concept of influence coefficient results from the presence of the disturbance potential which follows from the presence of the singularities. Formally, an influence coefficient is defined as the velocity induced at a field point by a unit strength singularity (be it a point singularity or a distributed singularity) placed anywhere within the flow field. Recall that equations 2.9 and 2.10 require the computation of the normal and tangential velocity components at all the elements control points. The normal components of velocities are essential in satisfying the flow tangency conditions, while the tangential components of velocities are necessary for satisfying the Kutta condition as well as computing the pressure distribution.

---

[3] This follows Giesing's notation where the velocity with respect to the airfoil frame is denoted as total velocity. While there is no misunderstanding in the steady flow, the notation actually refers to the relative velocity with respect to the airfoil moving frame of reference for the unsteady flow.

2. Notation for Influence Coefficient

The notation used in this documentation will be as for the notation for the single airfoil case with a slight modification to take into account the effects of the the second airfoil. As the influence coefficients are related to the geometry of the airfoil and their relative positions, it must, of necessity be computed with respect to the global frame of reference for the two airfoils' case.

For steady flow, the following influence coefficients are defined.

- A^n_{ij} : normal velocity component induced at the i^{th} control point by unit strength source distribution on the j^{th} panel.
- A^t_{ij} : tangential velocity component induced at the i^{th} control point by unit strength source distribution on the j^{th} panel.
- B^n_{ij} : normal velocity component induced at the i^{th} control point by unit strength vorticity distribution on the j^{th} panel.
- B^t : tangential velocity component induced at the i^{th} control point by unit strength vorticity distribution on the j^{th} panel.

where i and j denotes panel numbers and are defined as :

$i^{th} =$ 1, 2, ... n, n+1 ... 2n

$j^{th} =$ 1, 2, ... n, n+1 ... 2n

3. Computation of Influence Coefficient

A single source located on the j^{th} panel in the local frame of reference (see Figure 3) induces a total velocity of

$$V = \frac{1}{2\pi r} \tag{2.11}$$

where the component perpendicular to the inducing panel is

$$V^n = \frac{1}{2\pi r} \cdot \frac{h}{r} = \frac{1}{2\pi} \cdot \frac{h}{h^2 + s^2} \tag{2.12}$$

and the component parallel to the inducing panel is

$$V^s = \frac{1}{2\pi r} \cdot \left(-\frac{s}{r} \right) = -\frac{1}{2\pi} \cdot \frac{s}{h^2 + s^2} \tag{2.13}$$

For a distributed source panel on the j^{th} panel in the local frame of reference, we have

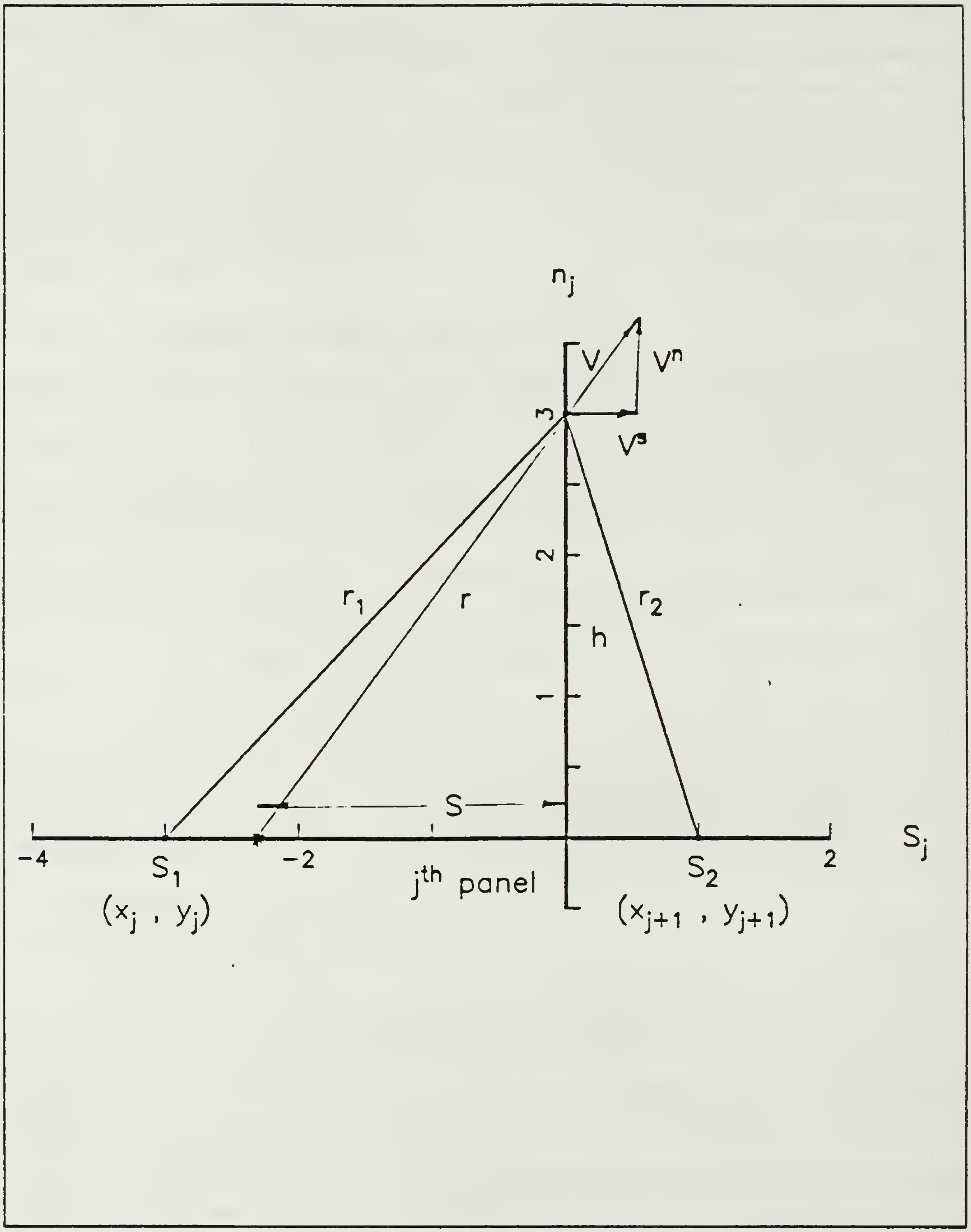


Figure 3. Influence Coefficient for Source Panel

$$V_{ij}^{n} = \int_{S_1}^{S_2} V_n(s)\, ds = \frac{1}{2\pi}\ FTAN \tag{2.14}$$

where

$$FTAN = tan^{-1} \frac{(xm_i - x_j)(ym_i - y_{j+1}) - (ym_i - y_j)(xm_i - x_{j+1})}{(xm_i - x_j)(xm_i - x_{j+1}) + (ym_i - y_j)(ym_i - y_{j+1})}$$

$$xm_i = 1/2\,(x_i + x_{i+1})$$

$$ym_i = 1/2\,(y_i + y_{i+1}) \tag{2.15}$$

$$V_{ij}^{s} = \int_{S_1}^{S_2} V_s(s)\, ds = -\frac{1}{2\pi}\ FLOG \tag{2.16}$$

where

$$FLOG = \ln \frac{(xm_i - x_{j+1})^2 + (ym_i - y_{j+1})^2}{(xm_i - x_j)^2 + (ym_i - y_j)^2} \tag{2.17}$$

Transforming the above to global frame of reference, we have

$$A_{ij}^{n} = V_{ij}^{n}\text{Cos}(\theta_i - \theta_j) - V_{ij}^{s}\text{Sin}(\theta_i - \theta_j) \tag{2.18}$$

$$A_{ij}^{t} = V_{ij}^{n}\text{Sin}(\theta_i - \theta_j) + V_{ij}^{s}\text{Cos}(\theta_i - \theta_j) \tag{2.19}$$

$$B_{ij}^{n} = -\,(V_{ij}^{n}\text{Cos}(\theta_i - \theta_j) - V_{ij}^{s}\text{Sin}(\theta_i - \theta_j)) \tag{2.20}$$

$$B_{ij}^{t} = V_{ij}^{n}\text{Sin}(\theta_i - \theta_j) - V_{ij}^{s}\text{Cos}(\theta_i - \theta_j) \tag{2.21}$$

For the Steady Flow code, we define

$$AAN(I,J) = A_{ij}^{n} = B_{ij}^{t} \tag{2.22}$$

$$BBN(I.J) = -\,A_{ij}^{t} = B_{ij}^{n} \tag{2.23}$$

E. NUMERICAL SOLUTION SCHEME

1. Rewriting the Boundary Condition

Though it is not critical for the steady flow problem to adopt any particular frame of reference, we will adopt the local frame of reference in satisfying the boundary condition for consistency with the treatment used in the unsteady case.

Even though the influence coefficients are defined in terms of the global frame of reference, the computation of the normal and tangential velocities of each panel is independent of the coordinate system used. Thus the normal and tangential velocities obtained with the global coordinates system would be the same as those obtained for the local coordinate system. Using the local coordinate system, the flow tangency condition of equation 2.9 is defined as follows :

$$(V^n)_i = \sum_{j=1}^{n} A^n_{ij} q_j + \sum_{j=n+1}^{2n} A^n_{ij} q_j + \gamma(1) \sum_{j=1}^{n} B^n_{ij} + \gamma(2) \sum_{j=n+1}^{2n} B^n_{ij} + V_\infty \operatorname{Sin}[\alpha(l) - \theta_i] = 0 \qquad (2.24)$$

where

$i = 1, 2 \ldots n \quad ; \; l = 1,$
$\; = n + 1 \ldots 2n \; ; \quad = 2$

The Kutta condition of eqn 2.10, in terms of influence coefficients, becomes for airfoil 1:

$$\sum_{j=1}^{n} A^t_{1j} q_j + \sum_{j=n+1}^{2n} A^t_{1j} q_j + \gamma(1) \sum_{j=1}^{n} B^t_{1j} + \gamma(2) \sum_{j=n+1}^{2n} B^t_{1j} + V_\infty \operatorname{Cos}[\alpha(1) - \theta_1] =$$

$$- \left\{ \sum_{j=1}^{n} A^t_{nj} q_j + \sum_{j=n+1}^{2n} A^t_{nj} q_j + \gamma(1) \sum_{j=1}^{n} B^t_{nj} + \gamma(2) \sum_{j=n+1}^{2n} B^t_{nj} + V_\infty \operatorname{Cos}[\alpha(1) - \theta_n] \right\} \qquad (2.25)$$

and for airfoil 2:

$$\sum_{j=1}^{n} A^t_{n+1,j} q_j + \sum_{j=n+1}^{2n} A^t_{n+1,j} q_j + \gamma(1) \sum_{j=1}^{n} B^t_{n+1,j} + \gamma(2) \sum_{j=n+1}^{2n} B^t_{n+1,j} + V_\infty \operatorname{Cos}[\alpha(2) - \theta_{n+1}] =$$

$$-\left\{\sum_{j=1}^{n} A^{t}_{2n,j} q_j + \sum_{j=n+1}^{2n} A^{t}_{2n,j} q_j + \gamma(1)\sum_{j=1}^{n} B^{t}_{2n,j} + \gamma(2)\sum_{j=n+1}^{2n} B^{t}_{2n,j} + V_{\infty}\,\mathrm{Cos}[\alpha(2)-\theta_{2n}]\right\} \quad (2.26)$$

2. Solving for the Strengths of Source and Vorticity Distribution

Equations 2.24, 2.25 and 2.26 can be written as a set of 2n+2 linear simultaneous equations with 2n+2 unknowns ($q_j, j = 1,2 \ldots 2n$ and $\gamma_l, l = 1,2$) and solved as for the single airfoil case. However, to make the routine consistent with the unsteady flow case, the following method is adopted.

The flow tangency condition can be rewritten explicitly for the q_j in terms of $\gamma(1)$, $\gamma(2)$ and the free stream constant term; that is

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,2n} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,2n} \\ a_{3,1} & a_{3,2} & a_{3,3} & \cdots & a_{3,2n} \\ \bullet & \bullet & \bullet & \cdots & \bullet \\ \bullet & \bullet & \bullet & \cdots & \bullet \\ \bullet & \bullet & \bullet & \cdots & \bullet \\ a_{2n,1} & a_{2n,2} & a_{2n,3} & \cdots & a_{2n,2n} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \bullet \\ \bullet \\ \bullet \\ q_{2n} \end{bmatrix} = \begin{bmatrix} a_{1,2n+1} \\ a_{2,2n+1} \\ a_{3,2n+1} \\ \bullet \\ \bullet \\ \bullet \\ a_{2n,2n+1} \end{bmatrix} \gamma(1) + \begin{bmatrix} a_{1,2n+2} \\ a_{2,2n+2} \\ a_{3,2n+2} \\ \bullet \\ \bullet \\ \bullet \\ a_{2n,2n+2} \end{bmatrix} \gamma(2) + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \bullet \\ \bullet \\ \bullet \\ b_{2n} \end{bmatrix}$$

(2.27)

Gauss Elimination is then used to solve for the q_j in terms of $\gamma(1)$, $\gamma(2)$ and the constant term. This gives

$$q_j = b1_j\,\gamma(1) + b2_j\,\gamma(2) + b3_j \quad j = 1,2 \ldots 2n \qquad (2.28)$$

Equation 2.28 is then substituted into the Kutta condition at the two trailing edges to form two linear simultaneous equation with two unknowns $\gamma(1)$ and $\gamma(2)$. Gauss Eimination is again used to solve for the Vorticity distribution $\gamma(1)$ and $\gamma(2)$ and these are then back substituted to solve for the q_j.

3. Computation of Velocity and Pressure Distribution

Once the q_j ($j = 1,2 \ldots 2n$) and γ_l ($l = 1,2$) are solved, the velocities at all the panel control points can be easily obtained. The normal velocity is given by

$$(V^n)_i = \sum_{j=1}^{n} A^n_{ij} q_j + \sum_{j=n+1}^{2n} A^n_{ij} q_j + \gamma(1) \sum_{j=1}^{n} B^n_{ij} + \gamma(2) \sum_{j=n+1}^{2n} B^n_{ij} + V_\infty \, \mathrm{Sin}[\, \alpha(l) - \theta_i \,] \tag{2.29}$$

while the tangential velocity is given by

$$(V^t)_i = \sum_{j=1}^{n} A^t_{ij} q_j + \sum_{j=n+1}^{2n} A^t_{ij} q_j + \gamma(1) \sum_{j=1}^{n} B^t_{ij} + \gamma(2) \sum_{j=n+1}^{2n} B^t_{ij} + V_\infty \, \mathrm{Cos}[\, \alpha(l) - \theta_i \,] \tag{2.30}$$

with

$i = 1,2 \ldots n \quad ; \; l = 1,$

$\quad = n+1 \ldots 2n \; ; \quad = 2$

The normal velocity will obviously satisfy equation 2.9 (used to check the code) showing that the flow tangency condition is satisfied while the the total velocity will be given by the tangential velocity.

$$V_{total} = (V^t)_i \, , \quad i = 1,2 \ldots 2n \tag{2.31}$$

where

$$(V^t)_i = -\sum_{j=1}^{n} B^n_{ij} q_j - \sum_{j=n+1}^{2n} B^n_{ij} q_j + \gamma(1) \sum_{j=1}^{n} A^n_{ij} + \gamma(2) \sum_{j=n+1}^{2n} A^n_{ij} + V_\infty \mathrm{Cos}[\, \alpha(l) - \theta_i \,] \tag{2.32}$$

Substituting equation 2.31 into 2.8 for C_p with $(V^t)_i$ defined as in equation 2.32, the pressure coefficient at the i^{th} control point is

$$(C_p)_i = 1 - (V^t)_i^2 \, , \quad i = 1,2 \ldots 2n \tag{2.33}$$

4. Computation of Forces and Moments

The two-dimensional aerodynamic lift (C_l), drag (C_d) and pitching moment (C_m) are calculated with respect to the global frame of reference. The moment coefficients are computed with respect to the respective leading edges. For the code, we have

$$C_l(l) = \sum_{i=1}^{n} (C_p)_i (X_{i+1} - X_i) \tag{2.34}$$

$$C_d(l) = -\sum_{i=1}^{n} (C_p)_i (Y_{i+1} - Y_i) \qquad (2.35)$$

$$C_m(l) = \sum_{i=1}^{n} (C_p)_i \{ (X_{i+1} - X_i) Xm_i + (Y_{i+1} - Y_i) Ym_i \} \qquad (2.36)$$

where $l = 1,2$ and X_i, X_{i+1}, Y_i, Y_{i+1}, Xm_i, Ym_i are defined as before.

III. UNSTEADY FLOW PROBLEM FORMULATION

A. GENERAL

The modification work for the unsteady flow is much more involved. It requires the following:

1. The establishment of five frames of reference viz one fixed inertia frame of reference (global), two moving local frames of reference[4] and two frozen local frames of reference.[5]
2. Reformulation of the two Kutta conditions which are coupled non-linearly. The solution requires an iterative procedure to compute for the two $\gamma(l)$. There are two possible solutions to the Kutta condition due to the quadratic nature of the equations. The solution which ensures that the product of the tangential velocities of the first and last panels of each airfoil is negative is accepted as the solution.
3. The creation of a new subroutine (SUBROUTINE NEWPOS) which transforms all coordinates in either of the two respective local frames of reference to the global frame of reference. This simplifies the definition of the airfoil, wake element and core vortices relative global geometries. The code requires that the airfoil be defined with respect to the respective frozen local frames of reference once only. Subsequent time dependent airfoil motion, wake panel behaviour and core vortices convection are computer generated.
4. The introduction of a more accurate method to obtain the velocity potential by integrating the velocity over smaller panels on the airfoil without having to store large arrays of influence coefficients which are not needed for satisfying the flow tangency condition.
5. Extension of the influence coefficient to include the effects of the second airfoil with its own peculiar wake. This also requires an introduction of an additional influence coefficient, that on the wake element due to the wake element from the other airfoil.

B. FRAMES OF REFERENCE

The inertia (global) and the two frozen local frames of reference at a specified time t_k are defined as for the steady flow case. The two moving local frames of reference have their x-y axes as for the frozen local frame of reference but this frame is moving with the airfoil.

---

[4] Moving local frames of reference are used to satisfy the flow tangency equation which simplifies to equation 2.9.

[5] Frozen local frames of reference are inertia frames (used in the steady flow case) of reference used to convect the core vortices of the previous time steps with respect to that time step frame of reference and subsequently transformed to the current time step frozen frame of reference.

C. UNSTEADY FLOW MODEL

1. Rigid Body Motion

The rigid body motion for the two airfoil system is an extension to the single airfoil system. Both airfoils are considered to have a mean velocity of $-V_\infty$, with a time dependent translational velocity of $-[U(l)_k\vec{i} + V(l)_k\vec{j}]$ and a rotational velocity $-\Omega(l)_k$ which can be in phase or out of phase; $\vec{i}$ and $\vec{j}$ are unit vectors in the respective local frames of reference and Ω is positive in the clockwise direction. The flow will be determined with respect to the moving local frame of reference. The flow tangency condition takes its simplest form in the moving frame of reference. The flow tangency condition seen from this frame of reference will satisfy equation 2.9. The unsteady stream velocity, V_{stream} is made up by the vector sum of a mean velocity V_∞, a time dependent translational velocity $[U(l)_k\vec{i} + V(l)_k\vec{j}]$ and a rotational velocity $\Omega(l)_k$ (Figure 4).

$$V_{stream} = V_\infty\{\, Cos[\,\alpha(l)_k\,]\,\vec{i} + Sin[\,\alpha(l)_k\,]\,\vec{j}\,\} + [\,U(l)_k\vec{i} + V(l)_k\vec{j}\,] + \Omega(l)_k\,(y\vec{i} - x\vec{j}\,) \qquad (3.1)$$

The disturbance potential, of necessity, is defined with respect to the inertia frame of reference, for it is only in this frame that the flow is irrotational. It is then transformed to the moving frame of reference for ease in treating the flow tangency condition. The disturbance potential is redefined to include the contributions from the wake panel and the core vortices from both airfoils.

$$\phi = \phi_s + \phi_v + \phi_w + \phi_{cv} \qquad (3.2)$$

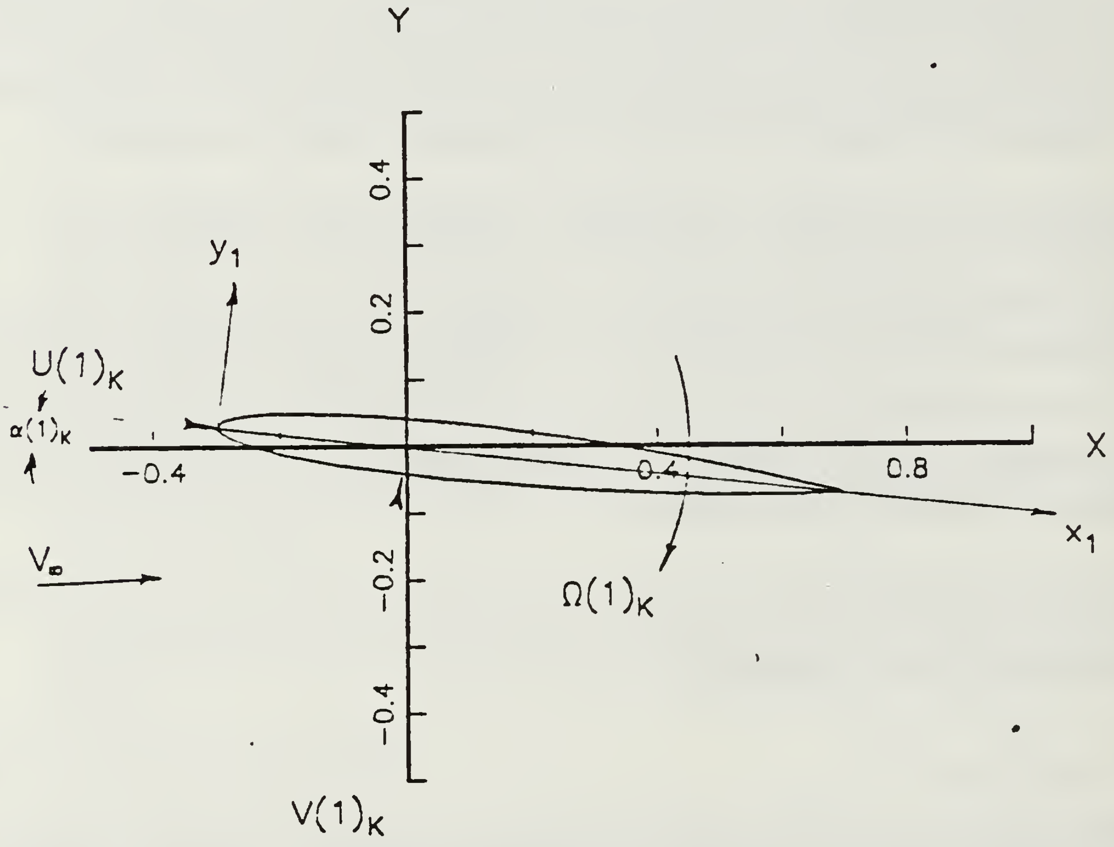
The total velocity is then written :

$$\vec{V}_{total} = \vec{V}_{stream} + \nabla\phi \qquad (3.3)$$

The unsteady Bernoulli's equation for the pressure coefficients on the airfoil surface is written with respect to the moving frame of reference. Giesing showed this to be written, in our notation as :

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho V_\infty^2} = \left(\frac{V_{stream}}{V_\infty}\right)^2 - \left(\frac{V_{total}}{V_\infty}\right)^2 - \frac{2}{V_\infty^2}\frac{\partial\phi}{\partial t} \qquad (3.4)$$

where V_{stream} and V_{total} are defined according to equations 3.1 and 3.3 respectively.



$$V_{stream} = V_\infty\{\,\mathrm{Cos}\,\alpha(1)_k\vec{i} + \mathrm{Sin}\,\alpha(1)_k\vec{j}\,\} + \{U(1)_k\vec{i} + V(1)_k\vec{j}\,\} + \Omega(1)_k(y\vec{i} - x\vec{j}\,)$$

Figure 4. Frames of Reference for Airfoil 1

2. Wake Model

Recall that the unsteady flow model requires an additional model to simulate the continuous shedding of vorticity into the trailing wake. The treatment of this vortex shedding process follows the approach of Basu and Hancock. The shed vorticity takes place through a small straight line wake element attached as an additional panel to the trailing edge of each airfoil with a uniform vorticity distribution $[\gamma_w(l)]_k$. This shed vorticity panel will be established if its length $\Delta(l)_k$ and inclination $\Theta(l)_k$ to the respective local frames of reference satisfy the Helmholtz theorem :

$$\Delta(l)_k[\gamma_w(l)]_k + \Gamma_k(l) = \Gamma_{k-1}(l) \tag{3.5}$$

$$\text{or } \Delta(l)_k[\gamma_w(l)]_k = \Gamma_{k-1}(l) - \Gamma_k(l) = SS(l)[\gamma(l)_{k-1} - \gamma(l)_k] \tag{3.6}$$

where SS is the perimeter of the airfoil and Γ_{k-1} and γ_{k-1} are respectively the total circulation and vorticity strength which are determined at the previous time step t_{k-1}.

At the next time step t_{k-1}, the shed vorticity panel will be detached from the trailing edge and will convect downstream as a concentrated free vortex with circulation $\Delta(l)_k[\gamma_w(l)]_k$ or $\Gamma_{k-1}(l) - \Gamma_k(l)$ at the resultant local velocity of the fluid particle. This wake convection process is illustrated in Figure 5 where the airfoils' subscripts are dropped without loss of generality.

In the code, the convection of the core vortices is broken into three steps:

1. The core vortices are first convected using the resultant absolute velocity with respect to the frozen local frame of reference of the previous time step.
2. This is followed by a transformation to the current frozen local frame of reference.
3. Finally this is transformed to the global frame of reference by using the new subroutine (SUBROUTINE NEWPOS).

3. Additional Boundary Conditions

The unsteady flow model has now introduced an additional boundary condition viz. the conservation of vorticity (equation 3.5 or 3.6) through the modeling of the wake. However, the introduction of the wake creates three additional unknowns for each airfoil, that is, $\gamma_w(l)_k$, $\Delta(l)_k$ and $\Theta(l)_k$. As such two additional conditions are required for each airfoil in order to solve the system. The approach suggested by Basu and Hancock is extended to the two airfoil case.

1. The wake panel is oriented in the direction of the local resultant velocity at the panel midpoint.

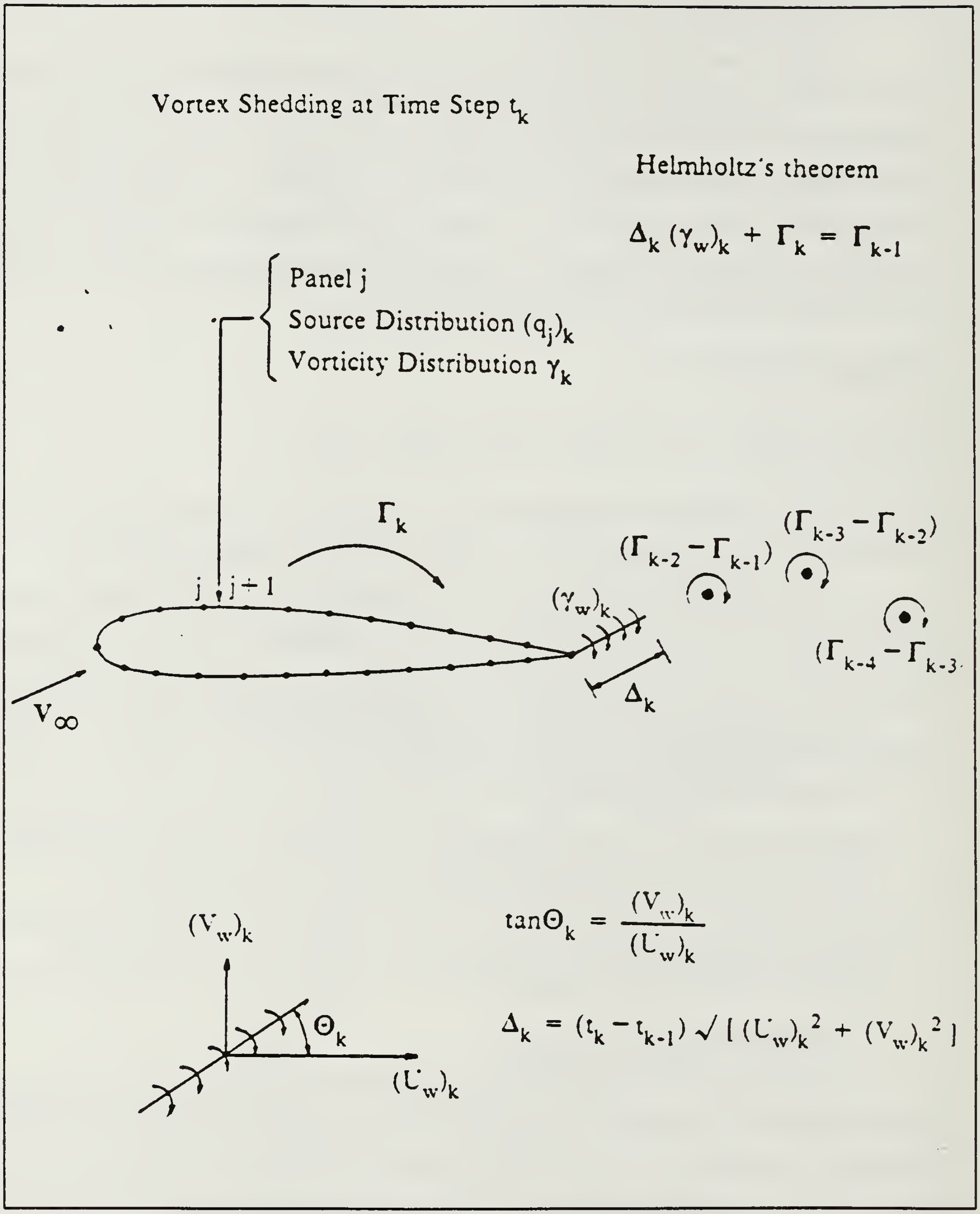

Vortex Shedding at Time Step t_k
Helmholtz's theorem
$\Delta_k (\gamma_w)_k + \Gamma_k = \Gamma_{k-1}$
Panel j
Source Distribution $(q_j)_k$
Vorticity Distribution γ_k
Γ_k
j
j+1
$(\Gamma_{k-3} - \Gamma_{k-2})$
$(\Gamma_{k-2} - \Gamma_{k-1})$
$(\gamma_w)_k$
$(\Gamma_{k-4} - \Gamma_{k-3})$
Δ_k
V_∞
$(V_w)_k$
$\tan\Theta_k = \frac{(V_w)_k}{(U_w)_k}$
Θ_k
$\Delta_k = (t_k - t_{k-1}) \sqrt{[(U_w)_k^2 + (V_w)_k^2]}$
$(U_w)_k$

Figure 5. Panel Methods Representation for Unsteady Flow

$$\tan\Theta(l)_k = \frac{[V_w^y(l)]_k}{[V_w^x(l)]_k} \tag{3.7}$$

where $[V_w^x(l)]_k$ and $[V_w^y(l)]_k$ are the x and y velocity components at the midpoint of the wake panel with respect to the frozen local frame of reference[6].

2. The length of the wake panel is propotional to the magnitude of the local resultant velocity at the panel midpoint and the step size of the time step.

$$\Delta(l)_k = (t_k - t_{k-1})\sqrt{([V_w^x(l)]_k)^2 + ([V_w^y(l)]_k)^2} \tag{3.8}$$

We now have all the necessary equations to solve for the system.

D. INFLUENCE COEFFICIENTS

As with the boundary condition, additional influence coefficients need to be defined as a result of the wake model. The definitions in the single airfoil case are thus extended as follows:

1. More A's and B's Influence Coefficients

Defining NP3 and NP4 as the panel number for the wake for the first and second airfoil respectively and h as a arbitrary core vortex, the following additional influence coefficient are required.

- $(B^n_{i,NP3})_k$: normal velocity component induced at the i^{th} panel control point by unit strength vorticity distribution on the wake panel of the first airfoil at time t_k.
- $(B^t_{i,NP3})_k$: tangential velocity component induced at the i^{th} panel control point by unit strength vorticity distribution on the wake panel of the first airfoil at time t_k.
- $(B^n_{i,NP4})_k$: normal velocity component induced at the i^{th} panel control point by unit strength vorticity distribution on the wake panel of the second airfoil at time t_k.
- $(B^t_{i,NP4})_k$ tangential velocity component induced at the i^{th} panel control point by unit strength vorticity distribution on the wake panel of the second airfoil at time t_k.
- $(A^x_{NP3,j})_k$: x velocity component induced at the first airfoil wake panel midpoint with respect to the frozen local frame of reference by unit strength source distribution on the j^{th} panel at time t_k.
- $(A^y_{NP3,j})_k$: y velocity component induced at the first airfoil wake panel midpoint with respect to the frozen local frame of reference by unit strength source distribution on the j^{th} panel at time t_k.
- $(A^x_{NP4,j})_k$: x velocity component induced at the second airfoil wake panel midpoint with respect to the frozen local frame of reference by unit strength source distribution on the j^{th} panel at time t_k.

---

[6] It is important to note that the velocities at the midpoint of the wake panels do not include the effect due to the vorticity distributed along itself but it does include the contribution due to the wake panel of the other airfoil.

- $(A^y_{NP4,j})_k$: y velocity component induced at the second airfoil wake panel midpoint with respect to the frozen local frame of reference by unit strength source distribution on the j^{th} panel at time t_k.
- $(B^x_{NP3,j})_k$: x velocity component induced at the first airfoil wake panel midpoint with respect to the frozen local frame of reference by unit strength vorticity distribution on the j^{th} panel at time t_k.
- $(B^y_{NP3,j})_k$: y velocity component induced at the first airfoil wake panel midpoint with respect to the frozen local frame of reference by unit strength vorticity distribution on the j^{th} panel at time t_k.
- $(B^x_{NP4,j})_k$: x velocity component induced at the second airfoil wake panel midpoint with respect to the frozen local frame of reference by unit strength vorticity distribution on the j^{th} panel at time t_k.
- $(B^y_{NP4,j})_k$: y velocity component induced at the second airfoil wake panel midpoint with respect to the frozen local frame of reference by unit strength vorticity distribution on the j^{th} panel at time t_k.
- $(A^x_{h,j})_k$: x velocity component induced at the h^{th} core vortex with respect to the frozen local frame of reference by unit strength source distribution on the j^{th} panel at time t_k.
- $(A^y_{h,j})_k$: y velocity component induced at the h^{th} core vortex with respect to the frozen local frame of reference by unit strength source distribution on the j^{th} panel at time t_k.
- $(B^x_{h,j})_k$: x velocity component induced at the h^{th} core vortex with respect to the frozen local frame of reference by unit strength vorticity distribution on the j^{th} panel at time t_k.
- $(B^y_{h,j})_k$: y velocity component induced at the h^{th} core vortex with respect to the frozen local frame of reference by unit strength vorticity distribution on the j^{th} panel at time t_k.
- $(B^x_{h,NP3})_k$: x velocity component induced at the h^{th} core vortex by unit strength vorticity distribution on the wake panel of the first airfoil at time t_k.
- $(B^y_{h,NP3})_k$: y velocity component induced at the h^{th} core vortex by unit strength vorticity distribution on the wake panel of the first airfoil at time t_k.
- $(B^x_{h,NP4})_k$: x velocity component induced at the h^{th} core vortex by unit strength vorticity distribution on the wake panel of the second airfoil at time t_k.
- $(B^y_{h,NP4})_k$: y velocity component induced at the h^{th} core vortex by unit strength vorticity distribution on the wake panel of the second airfoil at time t_k.

2. New C's Influence Coefficient

The single airfoil definition is extended to include the effects of the second airfoil.

- $(C^n_{i,m}(l))_k$: normal velocity component induced at the i^{th} panel control point by unit strength m^{th} core vortex at time t_k.

- $(C^t_{i,m}(l))_k$: tangential velocity component induced at the i^{th} panel control point by unit strength m^{th} core vortex in the wake of the l^{th} at time t_k.
- $(C^x_{NP3,m}(l))_k$: x velocity component with respect to the frozen local frame of reference induced at the first airfoil wake element by unit strength m^{th} core vortex in the wake of the l^{th} at time t_k.
- $(C^y_{NP3,m}(l))_k$: y velocity component with respect to the frozen local frame of reference induced at the first airfoil wake element by unit strength m^{th} core vortex in the wake of the l^{th} at time t_k.
- $(C^x_{NP4,m}(l))_k$: x velocity component with respect to the frozen local frame of reference induced at the second airfoil wake element by unit strength m^{th} core vortex in the wake of the l^{th} at time t_k.
- $(C^y_{NP4,m}(l))_k$: y velocity component with respect to the frozen local frame of reference induced at the second airfoil wake element by unit strength m^{th} core vortex in the wake of the l^{th} at time t_k.
- $(C^x_{h,m}(l))_k$: x velocity component with respect to the frozen local frame of reference induced at the h^{th} core vortex by unit strength m^{th} core vortex in the wake of the l^{th} at time t_k.
- $(C^y_{h,m}(l))_k$: y velocity component with respect to the frozen local frame of reference induced at the h^{th} core vortex by unit strength m^{th} core vortex in the wake of the l^{th} at time t_k.

3. Computation of the Time-Dependent Influence Coefficient

As for the single airfoil , the influence coefficients of the wake element $(B^x_{i,NP3})_k$,$(B^y_{i,NP3})_k$,$(B^x_{i,NP4})$ and $(B^y_{i,NP4})_k$ are computed as for the airfoil influence coefficients B^x_{ij} and B^y_{ij} with the subscripts NP3,NP4 replacing j.

The x and y velocity components for the respective frozen local frames of reference are obtained indirectly by first calculating for the global X and Y velocity. These global velocities are then transformed to the frozen frames through a simple relationship by the use of the respective angle of attack. It can be easily shown that the velocity with respect to the frozen frame of reference is related to the global velocity by the following relationship.

$$(V^x_{i,m})_k = [GV^x_{i,m} \cos \alpha(l)]_k - [GV^y_{i,m} \sin \alpha(l)]_k$$

$$(V^y_{i,m})_k = [GV^x_{i,m} \sin \alpha(l)]_k + [GV^y_{i,m} \cos \alpha(l)]_k \tag{3.9}$$

where V denotes a generic induced velocity with respect to the frozen frame and the precedent G denotes global velocities.

The global $(GA^x_{NP3,j})_k, (GA^y_{NP3,j})_k, (GA^x_{NP4,j})_k, \ldots (GB^y_{NP4,j})_k, (GA^x_{ij})_k, (GA^y_{ij})_k, (GB^x_{ij})_k$ and $(GB^y_{ij})_k$ are computed using equations 2.18 through 2.21 with θ_i set to zero and subscript i appropriately replaced.

The global C's coefficients will be computed identically as for the single airfoil case. The results are repeated here for the sake of completeness.

$$(GC^n_{im}(l))_k = -\frac{Cos[(\theta_i)_k - (\theta_m)_k]}{2\pi(r_{im})_k} \tag{3.10}$$

$$(GC^t_{im}(l))_k = -\frac{Sin[(\theta_i)_k - (\theta_m)_k]}{2\pi(r_{im})_k} \tag{3.11}$$

where :

$(r_{im})_k = \sqrt{[(xm_i - x_m)^2 + (ym_i - y_m)^2]}$

$xm_i = 1/2\,(x_i + x_{i-1})$

$ym_i = 1/2\,(y_i + y_{i-1})$

x_m = x coordinate of m^{th} core vortex at time t_k

y_m = y coordinate of m^{th} core vortex at time t_k

$\theta_i = tan^{-1}(\dfrac{y_{i-1} - y_i}{x_{i-1} - x_i})$

$(\theta_m)_k = tan^{-1}(\dfrac{ym_i - y_m}{xm_i - x_m})_k$

Also $(GC^x_{NP3,m}(l))_k$, $(GC^y_{NP3,m}(l))_k$, $(GC^x_{NP4,m}(l))_k$, $(GC^y_{NP4,m}(l))_k$, $(GC^x_{h,m}(l))_k$ and $(GC^y_{n,m}(l))_k$ are computed with equation 3.10 with θ_i set to zero and the subscript i appropriately replaced.

E. NUMERICAL SOLUTION SCHEME

1. The Flow Tangency Condition

The unsteady flow tangency equation for the two airfoil system is a simple extension of the single airfoil case.

$$\sum_{j=1}^{n}[A^n_{ij}(q_j)_k] + \sum_{j=n+1}^{2n}[A^n_{ij}(q_j)_k] + \gamma(1)_k\sum_{j=1}^{n}B^n_{ij} + \gamma(2)_k\sum_{j=n+1}^{2n}B^n_{ij} + [(\vec{V}_{stream})_i \bullet \vec{n}_i]_k$$

$$+ [\gamma_w(1)]_k(B^n_{i,NP3})_k + [\gamma_w(2)]_k(B^n_{i,NP4})_k +$$

$$\left\{\sum_{m=1}^{k-1}[(C^n_{im}(l))(\Gamma_{m-1}(l)-\Gamma_m(l))]\right\}_{l=1}+\left\{\sum_{m=1}^{k-1}[(C^n_{im}(l))(\Gamma_{m-1}(l)-\Gamma_m(l))]\right\}_{l=2} = 0$$

where i= 1,2 ... 2n

(3.12)

where i= 1,2 ... 2n and $\vec{V}_{stream}$ is evaluated by equation 3.1 at the i^{th} panel control point.

Rearranging the equation we can express the source strengths explicitly as a function of the two vorticity strengths and a constant term as in equation 2.28 where :

$$L.H.S. = \sum_{j=1}^{n}[A^n_{ij}(q_j)_k] + \sum_{j=n+1}^{2n}[A^n_{ij}(q_j)_k] \tag{3.13}$$

$$First\ R.H.S. = \gamma(1)_k\left\{\left(\frac{SS(1)}{\Delta(1)}\right)_k(B^n_{i,NP3})_k - \sum_{j=1}^{n}(B^n_{ij})_k\right\} \tag{3.14}$$

$$Second\ R.H.S. = \gamma(2)_k\left\{\left(\frac{SS(2)}{\Delta(2)}\right)_k(B^n_{i,NP4})_k - \sum_{j=n+1}^{2n}(B^n_{ij})_k\right\} \tag{3.15}$$

$$Third\ R.H.S. = -[(\vec{V}_{stream})_i \bullet \vec{n}_i]_k - \gamma(1)_{k-1}\left(\frac{SS(1)}{\Delta(1)}\right)_k(B^n_{i,NP3})_k -$$

$$\gamma(2)_{k-1}\left(\frac{SS(2)}{\Delta(2)}\right)_k(B^n_{i,NP4})_k$$

$$-\left\{\sum_{m=1}^{k-1}[(C^n_{im}(l))(\Gamma_{m-1}(l)-\Gamma_m(l))]\right\}_{l=1}-\left\{\sum_{m=1}^{k-1}[(C^n_{im}(l))(\Gamma_{m-1}(l)-\Gamma_m(l))]\right\}_{l=2} \tag{3.16}$$

where i = 1,2 ... 2n

For the code, the above is implemented in a $2N \times (2N+3)$ matrix in SUBROUTINE COEF and is subsequently solved by GAUSS elimination to obtain an explicit expression for the source strength in terms of the vorticity distribution and a constant term as in the steady flow case as follows :

$$q_j = b1_j \cdot \gamma(1) + b2_j \cdot \gamma(2) + b3_j \quad , \quad j = 1,2 \ldots 2n \tag{3.17}$$

where

$b1_j$: part of the source strength which depends on circulation $\gamma(1)_k$.

$b2_j$: part of the source strength which depends on circulation $\gamma(2)_k$.

$b3_j$: part of the source strength which is independent of the circulation.

Equation 3.17 is critical in the simplification of the treatment for the non-linear coupled Kutta condition.

2. The Kutta Condition

For the unsteady flow, the Kutta condition can be written as the following :

For the first airfoil

$$\begin{aligned}(V_1^t)_k^2 - (V_n^t)_k^2 &= 2[\,\frac{\partial}{\partial t}(\phi_n - \phi_1)\,]_k = 2(\frac{\partial \Gamma(1)}{\partial t})_k \\ &= 2SS(1)\frac{\gamma_k(1) - \gamma_{k-1}(1)}{t_k - t_{k-1}}\end{aligned} \tag{3.18}$$

For the second airfoil

$$\begin{aligned}(V_{n+1}^t)_k^2 - (V_{2n}^t)_k^2 &= 2[\,\frac{\partial}{\partial t}(\phi_{2n} - \phi_{n+1})\,]_k = 2(\frac{\partial \Gamma(2)}{\partial t})_k \\ &= 2SS(2)\frac{\gamma_k(2) - \gamma_{k-1}(2)}{t_k - t_{k-1}}\end{aligned} \tag{3.19}$$

The tangential velocity for the first panel can be written thus:

$$\begin{aligned}V_1^t = &-\sum_{j=1}^{2n}(B_{1j}^n q_j)_k + \gamma(1)_k\sum_{j=1}^{n}A_{1j}^n + + \gamma(2)_k\sum_{j=n+1}^{2n}A_{1j}^n + \\ &\frac{SS(1)}{\Delta(1)}A_{1,NP3}^n[\gamma_{k-1}(1) - \gamma_k(1)] + \frac{SS(2)}{\Delta(2)}A_{1,NP4}^n[\gamma_{k-1}(2) - \gamma_k(2)] + [(\vec{V}_{stream})_i \cdot \vec{t}_i]_k + \\ &\left\{\sum_{m=1}^{k-1}[(C_{1m}^t(l))(\Gamma_{m-1}(l) - \Gamma_m(l))]\right\}_{l=1} + \left\{\sum_{m=1}^{k-1}[(C_{1m}^t(l))(\Gamma_{m-1}(l) - \Gamma_m(l))]\right\}_{l=2}\end{aligned} \tag{3.20}$$

Substituting equation 3.17 into 3.20, the following simplified relation for V_1^t is obtained.

$$V_1^t = D1_1\gamma(1)_k + D2_1\gamma(2)_k + D3_1 \tag{3.21}$$

where

$$D1_1 = -\sum_{j=1}^{2n} B_{1j}^n b1_1 + \sum_{j=1}^{n} A_{1j}^n - \frac{SS(1)}{\Delta(1)} A_{1,NP3}^n$$

$$D2_1 = -\sum_{j=1}^{2n} B_{1j}^n b2_1 + \sum_{j=n+1}^{2n} A_{1j}^n - \frac{SS(2)}{\Delta(2)} A_{1,NP4}^n$$

$$D3_1 = -\sum_{j=1}^{2n} B_{1j}^n b3_1 + \frac{SS(1)}{\Delta(1)} A_{1,NP3}^n \gamma_{k-1}(1) + \frac{SS(2)}{\Delta(2)} A_{1,NP4}^n \gamma_{k-1}(2) + [(\vec{V}_{stream})_1 \bullet \vec{t}_1]_k +$$

$$+ \left\{\sum_{m=1}^{k-1} [(C_{1m}^t(l))(\Gamma_{m-1}(l) - \Gamma_m(l))]\right\}_{l=1} + \left\{\sum_{m=1}^{k-1} [(C_{1m}^t(l))(\Gamma_{m-1}(l) - \Gamma_m(l))]\right\}_{l=2} \tag{3.22}$$

Similar expressions can be obtained for the other trailing edge panels as follows :

$$V_n^t = D1_n\gamma(1)_k + D2_n\gamma(2)_k + D3_n \tag{3.23}$$

$$V_{n+1}^t = D1_{n+1}\gamma(1)_k + D2_{n+1}\gamma(2)_k + D3_{n+1} \tag{3.24}$$

$$V_{2n}^t = D1_{2n}\gamma(1)_k + D2_{2n}\gamma(2)_k + D3_{2n} \tag{3.25}$$

where the coefficients D1, D2, D3 are obtained as per equation 3.22. We require the square of the tangential velocity for the Kutta condition

$$\begin{aligned}(V_1^t)^2 &= (D1_1\gamma(1)_k + D2_1\gamma(2)_k + D3_1)^2 \\ &= D1_1^2\gamma(1)_k^2 + D2_1^2\gamma(2)_k^2 + D3_1^2 \\ &+ 2D1_1D2_1\gamma(1)_k\gamma(2)_k + 2D1_1D3_1\gamma(1)_k + 2D2_1D3_1\gamma(2)_k\end{aligned} \tag{3.26}$$

$$\begin{aligned}(V_n^t)^2 &= (D1_n\gamma(1)_k + D2_n\gamma(2)_k + D3_n)^2 \\ &= D1_n^2\gamma(1)_k^2 + D2_n^2\gamma(2)_k^2 + D3_n^2 \\ &+ 2D1_nD2_n\gamma(1)_k\gamma(2)_k + 2D1_nD3_n\gamma(1)_k + 2D2_nD3_n\gamma(2)_k\end{aligned} \tag{3.27}$$

For the first airfoil equations 3.26 and 3.27 are then substituted into 3.18 to give, for the code:

$$\begin{aligned} & AAA(1)\gamma(1)_k^2 + BBB(1)\gamma(2)_k^2 + CCC(1)\gamma(1)_k + \\ & DDD(1)\gamma(2)_k + EEE(1)\gamma(1)_k\gamma(2)_k + FFF(1) \quad = \quad 0 \end{aligned} \tag{3.28}$$

where

$$AAA(1) = D1_1^2 - D1_n^2$$

$$BBB(1) = D2_1^2 - D2_n^2$$

$$CCC(1) = 2[D1_1 \bullet D3_1 - D1_n \bullet D3_n - \frac{SS(1)}{t_k - t_{k-1}}]$$

$$DDD(1) = 2[D2_1 \bullet D3_1 - D2_n \bullet D3_n]$$

$$EEE(1) = 2[D1_1 \bullet D2_1 - D1_n \bullet D2_n]$$

$$FFF(1) = D3_1^2 - D3_n^2 + 2(\frac{SS(1)\gamma_{k-1}(1)}{t_k - t_{k-1}}) \tag{3.29}$$

A similar set of equation can be obtained for the second airfoil as follows:

$$\begin{aligned} & AAA(2)\gamma(1)_k^2 + BBB(2)\gamma(2)_k^2 + CCC(2)\gamma(1)_k + \\ & DDD(2)\gamma(2)_k + EEE(2)\gamma(1)_k\gamma(2)_k + FFF(2) \quad = \quad 0 \end{aligned} \tag{3.30}$$

where

$$AAA(2) = D1_{n+1}^2 - D1_{2n}^2$$

$$BBB(2) = D2_{n+1}^2 - D2_{2n}^2$$

$$CCC(2) = 2[D1_{n+1} \bullet D3_{n+1} - D1_{2n} \bullet D3_{2n}]$$

$$DDD(2) = 2[D2_{n+1} \bullet D3_{n+1} - D2_{2n} \bullet D3_{2n} - \frac{SS(2)}{t_k - t_{k-1}}]$$

$$EEE(2) = 2[D1_{n+1} \bullet D2_{n+1} - D1_{2n} \bullet D2_{2n}]$$

$$FFF(2) = D3^2_{n+1} - D3^2_{2n} + 2\left(\frac{SS(2)\gamma_{k-1}(2)}{t_k - t_{k-1}}\right) \tag{3.31}$$

The above Kutta equations are non-linear and coupled. To linearize it, we adopt Newton's method, in which the initial iteration is linearized about the values of the previous time step:

$$\gamma(1)^{\kappa}_{k} = \gamma(1)^{\kappa-1}_{k-1} + \delta\gamma(1)^{\kappa}_{k} \Rightarrow [\gamma(1)^{\kappa}_{k}]^2 = [\gamma(1)^{\kappa}_{k-1}]^2 + 2\gamma(1)^{\kappa-1}_{k-1}\delta\gamma(1)^{\kappa}_{k}$$

$$\gamma(2)^{\kappa}_{k} = \gamma(2)^{\kappa-1}_{k-1} + \delta\gamma(2)^{\kappa}_{k} \Rightarrow [\gamma(2)^{\kappa}_{k}]^2 = [\gamma(2)^{\kappa}_{k-1}]^2 + 2\gamma(2)^{\kappa-1}_{k-1}\delta\gamma(2)^{\kappa}_{k} \tag{3.32}$$

where:

$$\delta\gamma(1)^{\kappa}_{k} \ll \gamma(1)^{\kappa-1}_{k} \quad ; \quad \gamma(1)^{0}_{k} = \gamma(1)_{k-1}$$

$$\delta\gamma(2)^{\kappa}_{k} \ll \gamma(2)^{\kappa-1}_{k} \quad ; \quad \gamma(2)^{0}_{k} = \gamma(2)_{k-1}$$

with κ denoting the iteration counter. After dropping quadratic terms of $\delta\gamma$ and collecting like terms the Kutta condition for the first airfoil simplifies to the following:

$$\begin{aligned}
&\{2AAA(1)\gamma(1)_{k-1} + CCC(1) + EEE(1)\gamma(2)_{k-1}\}\delta\gamma(1)_k + \\
&\{2BBB(1)\gamma(2)_{k-1} + DDD(1) + EEE(1)\gamma(1)_{k-1}\}\delta\gamma(2)_k + \\
&AAA(1)\gamma(1)^2_{k-1} + BBB(1)\gamma(2)^2_{k-1} + CCC(1)\gamma(1)_{k-1} + \\
&DDD(1)\gamma(2)_{k-1} + EEE(1)\gamma(1)_{k-1}\gamma(2)_{k-1} + FFF(1) = 0
\end{aligned} \tag{3.33}$$

Similarly the linearised Kutta condition for the second airfoil can be written:

$$\begin{aligned}
&\{2AAA(2)\gamma(1)_{k-1} + CCC(2) + EEE(2)\gamma(2)_{k-1}\}\delta\gamma(1)_k + \\
&\{2BBB(2)\gamma(2)_{k-1} + DDD(2) + EEE(2)\gamma(1)_{k-1}\}\delta\gamma(2)_k + \\
&AAA(2)\gamma(1)^2_{k-1} + BBB(2)\gamma(2)^2_{k-1} + CCC(2)\gamma(1)_{k-1} + \\
&DDD(2)\gamma(2)_{k-1} + EEE(2)\gamma(1)_{k-1}\gamma(2)_{k-1} + FFF(2) = 0
\end{aligned} \tag{3.34}$$

The above two linear equations with two unknowns $\delta\gamma(1)_k$ and $\delta\gamma(2)_k$ can now be easily solved with the same Gauss routine. The results are then back substituted into equations 3.32. This procedure is repeated until the corrections $\delta\gamma(1)_k^r$ and $\delta\gamma(2)_k^r$ are less than a prescribed tolerance.

3. Convection of the Wake

The resultant velocities of all core vortices are calculated using the frozen local frame of reference. This is resolved into components at the h^{th} core vortex as follows:

$$\begin{aligned}(U_h)_k = &\sum_{j=1}^{n}[A_{hj}^{x}(q_j)]_k + \sum_{j=n+1}^{2n}[A_{hj}^{x}(q_j)]_k + \gamma(1)_k\sum_{j=1}^{n}(B_{hj}^{x})_k + \gamma(2)_k\sum_{j=n+1}^{2n}(B_{hj}^{x})_k + [(\vec{V}_\infty)_h \bullet \vec{i}\,]_k \\ &+ [\gamma_w(1)]_k(B_{h,NP3}^{x})_k + [\gamma_w(2)]_k(B_{h,NP4}^{x})_k \\ &+ \left\{\sum_{\substack{m=1\\ m\neq h}}^{k-1}[(C_{hm}^{x}(l))_k(\Gamma_{m-1}(l) - \Gamma_m(l))]\right\}_{l=1} + \left\{\sum_{\substack{m=1\\ m\neq h}}^{k-1}[(C_{hm}^{x}(l))_k(\Gamma_{m-1}(l) - \Gamma_m(l))]\right\}_{l=2}\end{aligned} \tag{3.35}$$

$$\begin{aligned}(V_h)_k = &\sum_{j=1}^{n}[A_{hj}^{y}(q_j)]_k + \sum_{j=n+1}^{2n}[A_{hj}^{y}(q_j)]_k + \gamma(1)_k\sum_{j=1}^{n}(B_{hj}^{y})_k + \gamma(2)_k\sum_{j=n+1}^{2n}(B_{hj}^{y})_k + [(\vec{V}_\infty)_h \bullet \vec{j}\,]_k \\ &+ [\gamma_w(1)]_k(B_{h,NP3}^{y})_k + [\gamma_w(2)]_k(B_{h,NP4}^{y})_k \\ &+ \left\{\sum_{\substack{m=1\\ m\neq h}}^{k-1}[(C_{hm}^{y}(l))_k(\Gamma_{m-1}(l) - \Gamma_m(l))]\right\}_{l=1} + \left\{\sum_{\substack{m=1\\ m\neq h}}^{k-1}[(C_{hm}^{y}(l))_k(\Gamma_{m-1}(l) - \Gamma_m(l))]\right\}_{l=2}\end{aligned} \tag{3.36}$$

The location of the core vortices at the new time step is then computed with respect to the current time step frozen local frame of reference and then transformed to the new time step frozen local frame of reference. These coordinates are subsequently transformed to global coordinates so as to facilitate the calculation of the influence coefficient.

4. Disturbance Potential and Pressure Distribution

The concept of disturbance potential introduced in the steady flow has played a vital role in that it facilitates the synthesis of the flow field from simple flow field by simple superposition of the various singularities contributions. However, there has not been a requirement to solve directly for the disturbance potential, as our interest was on the disturbance induced velocity which is the spatial derivative of the disturbance potential. In our solutions, the concept of influence coefficient has allowed a direct evaluation of the disturbance velocity thus nullifying the requirements of obtaining the disturbance potential.

The treatment for the unsteady flow, though it follows the same procedure as the steady flow case, viz the influence coefficient to obtain the disturbance velocity, still requires the disturbance potential for the computation of the pressure coefficient as can be seen in equation 3.4. Rewriting, we have:

$$(C_{p_i})_k = [(\frac{V_{stream}}{V_\infty})_i]_k^2 - [(\frac{V_{total}}{V_\infty})_i]_k^2 - \frac{2}{V_\infty^2}\frac{(\phi_i)_k - (\phi_i)_{k-1}}{t_k - t_{k-1}} \quad i = 1,2 \ldots 2n \qquad (3.37)$$

where V_{stream} and V_{total} are defined according to equations 3.1 and 3.3 respectively.

From equation 3.37 we have written the rate of change of ϕ by a backward finite difference approximation. This simplifies our iteration procedures tremendously as the ϕ from the previous time step exists at the current computation. The computation of the disturbance potenial ϕ is obtained through two steps. The difference in potential ϕ from upstream at infinity to the leading edge is computed and is then combined with the difference in the potential from the leading edge to the panel's of interest control point.

The present approach differs from the original single airfoil approach in that the velocity potential along the airfoil surface is computed via a finer grid using Gaussian quadrature[7]. This modification improves the results[8] but the cost of doing it is a longer computation time. The definition of infinity has also been set to 100 chord lengths in comparison to 10 chord lengths used in the original code. However, the number of computation points and the first panel length from the leading edge upstream have not been changed.

For completeness, the computation of the disturbance potential from the leading edge to infinity is included here for both airfoils. It is essentially the same as the single airfoil case except that the disturbance potential is now computed relative to the global frame of reference instead of the airfoil fixed frame of reference as used in the original code. We begin by selecting a straight line extending upstream in the direction parallel to V_∞. The length of the line is set at 100 chord lengths. This line is divided into z panels with the first element at the leading edge set equal to the single airfoil case for the purpose of ensuring that the panel size is comparable to the airfoil panel size. The panel size is then subsequently increased to take advantage of the inversely decaying induced velocities at larger distances. Using subscript f to denote these panel mid-points, we define the following influence coefficients:

---

[7] Each panel is subdivided into 4 additional sub-panels and the tangential velocity is computed and integated over these smaller panels with a weighting function to get the disturbance potential.

[8] One can check the improvement by computing the $\gamma(s)$ obtained through the difference between the trailing edges and compares them with that obtained through the Kutta condition.

- $(A^n_{f,j})_k$: normal velocity component induced at the f^{th} panel control point by unit strength source distribution on the j^{th}panel at time t_k.
- $(A^t_{f,j})_k$: tangential velocity component induced at the f^{th} panel control point by unit strength source distribution on the j^{th}panel at time t_k.
- $(B^n_{f,j})_k$: normal velocity component induced at the f^{th} panel control point by unit strength vorticity distribution on the j^{th}panel at time t_k.
- $(B^t_{f,j})_k$: tangential velocity component at the f^{th} panel control point by unit strength vorticity distribution on the j^{th}panel at time t_k.
- $(B^n_{f,NP3})_k$: normal velocity component induced at the f^{th} panel control point by unit strength vorticity distribution on the wake panel of the first airfoil at time t_k.
- $(B^t_{f,NP3})_k$: tangential velocity component induced at the f^{th} panel control point by unit strength vorticity distribution on the wake panel of the first airfoil at time t_k.
- $(B^n_{f,NP4})_k$: normal velocity component induced at the f^{th} panel control point by unit strength vorticity distribution on the wake panel of the second airfoil at time t_k.
- $(B^t_{f,NP4})_k$ tangential velocity component induced at the f^{th} panel control point by unit strength vorticity distribution on the wake panel of the second airfoil at time t_k.
- $(C^n_{f,m}(l))_k$: normal velocity component induced at the f^{th} panel control point of the l^{th} by unit strength m^{th} core vortex at time t_k.
- $(C^t_{f,m}(l))_k$: tangential velocity component induced at the f^{th} panel control point of the l^{th} by unit strength m^{th} core vortex at time t_k.

The tangential velocity at the f^{th} panel written as for the code is then:

$$(V^t_f)_k = -\sum_{j=1}^{2n}(B^n_{fj}q_j)_k + \gamma(1)_k\sum_{j=1}^{n}(A^n_{fj})_k + + \gamma(2)_k\sum_{j=n+1}^{2n}(A^n_{fj})_k +$$

$$\frac{SS(1)}{\Delta(1)}(A^n_{f,NP3})_k[\gamma_{k-1}(1) - \gamma_k(1)] + \frac{SS(2)}{\Delta(2)}(A^n_{f,NP4})_k[\gamma_{k-1}(2) - \gamma_k(2)] +$$

$$\left\{\sum_{m=1}^{k-1}[(C^t_{fm}(l))_k(\Gamma_{m-1}(l) - \Gamma_m(l))]\right\}_{l=1} + \left\{\sum_{m=1}^{k-1}[(C^t_{fm}(l))_k(\Gamma_{m-1}(l) - \Gamma_m(l))]\right\}_{l=2} \qquad (3.38)$$

valid for f= 1,2 ... z. The disturbance potential at the airfoil leading edge is the sum of the products of the disturbance induced velocity at each panel and the panel length .

$$(\phi_{\mathrm{le}})_k = -\sum_{f=1}^{z}[(V_f^t)_k \bullet (X_{f+1} - X_f)] \tag{3.39}$$

The integral over the airfoil surface, as stated before, is now done over a finer grid. This requires the computation of the disturbance induced velocity over the smaller grids within each panels. Defining the total finer grid points in one panel by P[9], we define first the refined influence coefficient

$(A^t_{ijp})_k$

tangential velocity induced at the i^{th} panel p^{th} node due to unit strength source distribution on the j^{th} panel at time t_k

The other refined influence coefficients have the same definition. The tangential component of the disturbance induced velocity at the i^{th} panel m^{th} node is then

$$\begin{aligned}(V^t_{ip})_k = & -\sum_{j=1}^{2n}(B^n_{ijp}q_j)_k + \gamma(1)_k\sum_{j=1}^{n}A^n_{ijp} + \gamma(2)_k\sum_{j=n+1}^{2n}A^n_{ijp} + \\ & \frac{SS(1)}{\Delta(1)}A^n_{i,NP3,p}[\gamma_{k-1}(1) - \gamma_k(1)] + \frac{SS(2)}{\Delta(2)}A^n_{i,NP4,p}[\gamma_{k-1}(2) - \gamma_k(2)] + \\ & \left\{\sum_{m=1}^{k-1}[(C^t_{imp}(l))(\Gamma_{m-1}(l) - \Gamma_m(l))]\right\}_{l=1} + \left\{\sum_{m=1}^{k-1}[(C^t_{imp}(l))(\Gamma_{m-1}(l) - \Gamma_m(l))]\right\}_{l=2}\end{aligned} \tag{3.40}$$

which is valid for i= 1,2 ... 2n ; p= 1,2 ... P. Performing the line integration by summation, the disturbance potential at the i^{th} nodal point is then

$$\begin{aligned}(\phi_{node\ i})_k &= (\phi_{\mathrm{le}})_k + \sum_{i=i_{\mathrm{le}}}^{i-1}\sum_{p=1}^{P}[(V_{ip})_k r_{j,j+1} W_p], \quad for\ n > i \geq i_{\mathrm{le}} \\ &= (\phi_{\mathrm{le}})_k - \sum_{i=i_i}^{i_{\mathrm{le}}-1}\sum_{p=1}^{P}[(V_{ip})_k r_{j,j+1} W_p], \quad for\ n > i \geq i_{\mathrm{le}}\end{aligned} \tag{3.41}$$

---

[9] This includes the end points.

where $r_{j,j-1}$ denotes the panel length and W_p the Gaussian quadrature weighting function.

$$r_{j,j+1} = \sqrt{(X_{j+1} - X_j)^2 + (Y_{j+1} - Y_j)^2} \qquad (3.42)$$

Finally the disturbance potential at the i^{th} panel control point is :

$$(\phi_i)_k = 1/2[\,(\phi_{node\,i})_k + (\phi_{node\,i+1})_k] \;,\quad i = 1,2 \ldots 2n \qquad (3.43)$$

5. Computation of Forces and Moments

The C_l, C_d and C_m about the leading edge of each airfoil are calculated in exactly the same way as it is done for the steady flow problem by integrating the pressure distribution (See section D-4 of Chapter 2).

IV. DESCRIPTION OF COMPUTER CODE USPOTF2

A. PROGRAM STRUCTURES AND CAPABILITIES

1. Restrictions and Limitations

The restrictions and limitations listed in the single airfoil documentation [Ref. 1] still apply for the two airfoil case. Some efforts were made to optimise storage requirements versus computational repetitions; however as in the original code there is still much room for improvement. Krainer [Ref. 6] has improved the original code by combining subroutines and reducing repeated computation by introducing additional common variables. Some of his improvements have been added to this program.

The computer system used is the Naval Postgraduate School IBM 3033AP. The current program fixes the maximum number of airfoil panels to 200[10] and the maximum allowable time steps to 200. The computer currently has to be run with a minimum storage requirement of 2 Mbytes. A detailed computing time study for the program is not undertaken, for it not only changes with the number of nodal points selected but also with the time step increment as this has an effect on the rate of convergence for the wake panel iteration. An order of magnitude is given for one case run to give an appreciation of the computing time required. The system currently requires a total CPU run time of 200 seconds using an optimising compiler for a step input with a 0.025 non-dimensionalised time increment for 26 time steps.

At present, the program can run for two airfoils set at arbitrary distance and at different angles of attack undergoing any of the following motions:

1. In-phase and out-of-phase Step Input
2. In-phase and out-of-phase Modified Ramp Input
3. In-phase and out-of-phase Translational Harmonic Oscillation
4. In-phase and out-of-phase Rotational Harmonic Oscillation
5. Sharp Edge Gust Field Penetration

2. Current Structures of USPOTF2 Main Program

The flow logic for USPOTF2 (Unsteady Potential Flow for 2 airfoils) is illustrated in Figure 6. It is essentially divided into three major modules, namely, the input

---

[10] These are combined total panels for both airfoils.

- problem setup, Steady flow solution with its associated output and the Unsteady flow solution with its associated output.

The first module includes subroutines INDATA, SETUP, BODY and NACA45. The main functions of this module are:

1. Set up the problem formulation by reading in the necessary values and flag setting from filecode 1.
2. If the flag for a NACA 4-digit or 5-digit of type 230XX is set, this module calls subroutines BODY and NACA45 to compute the local panel coordinates.
3. If the flag for a NACA 4-digit or 5-digit of type 230XX is not set, this module then proceeds to read in the local panel coordinates from filecode 1.
4. The local slopes and the airfoil perimeters are then calculated in preparation for the next module.

The second module calculates the Steady flow solution. It calls subroutines NEWPOS, INFL, COEF, GAUSS, KUTTA, VELDIS and FANDM. The main functions of this module are:

1. Transform the local airfoil coordinates into global coordinates and compute the influence coefficients.
2. Set up flow tangency equation as per equation 2.27 and solve for the source strengths (q_j) as a function of the vorticity strengths [$\gamma(l)$] and a constant part where j = 1,2 ... 2n and $l = 1,2$.
3. Set up and solve the Kutta condition as per equations 2.25 and 2.26 for the vorticity strengths and back substitute to get the source strengths.
4. Compute the total tangential velocity in the moving frame and compute the disturbance potential (ϕ_i) and pressure coefficient [$(C_p)_i$] (i = 1,2 ... 2n).
5. Finally, compute the aerodynamic coefficients of forces and moments -- C_l, C_d, C_m.
6. This program can terminate in this module after all the steady flow parameters are obtained without necessarily running the Unsteady flow code.

The third module calculates the Unsteady flow solution. It calls subroutines NEWPOS, INFL, COEF, GAUSS, KUTTA, TEWAK, PRESS, FANDM and CORVOR. The main functions of this module are:

1. For the particular unsteady motion, to compute the initial time step and new airfoil orientation with its associated local body velocities.
2. Introduce the wake panel and assume an initial length and orientation to begin iterations.
3. Transform all local coordinates and panel slopes to global coordinates and global panel slopes.

4. Update influence coefficient and set up flow tangency equation.
5. Solve for the source strengths in terms of the vorticity strengths and the constant part.
6. Invoke the required Kutta condition and solve for the vorticity strengths[11]; back substitute to get the source strengths.
7. Update the local velocities of the wake element and compute new length and orientation --- check for convergence.
8. If wake element velocities have not converged, iterate with the new wake element geometries until convergence[12].
9. Compute the total velocity, disturbance potential and the pressure coefficient.
10. Compute the aerodynamic lift, drag and moment coefficient.
11. Adjust time step either through an interative input[13] or by a automatic time increment.
12. Compute resultant local velocities of the core vortices.
13. Convect the core vortices by the procedure described in Chapter III section E-3

---

[11] There are two possible solutions; only the vorticity distribution that ensures that the product of the relative tangential velocities of the upper and lower panels of the trailing edge is negative is accepted as solution.

[12] The convergence criterion is user specified through input data for TOL.

[13] This is accomplished by setting TADJ to be non-zero and is intended for use in conjunction with the viscous flow program.

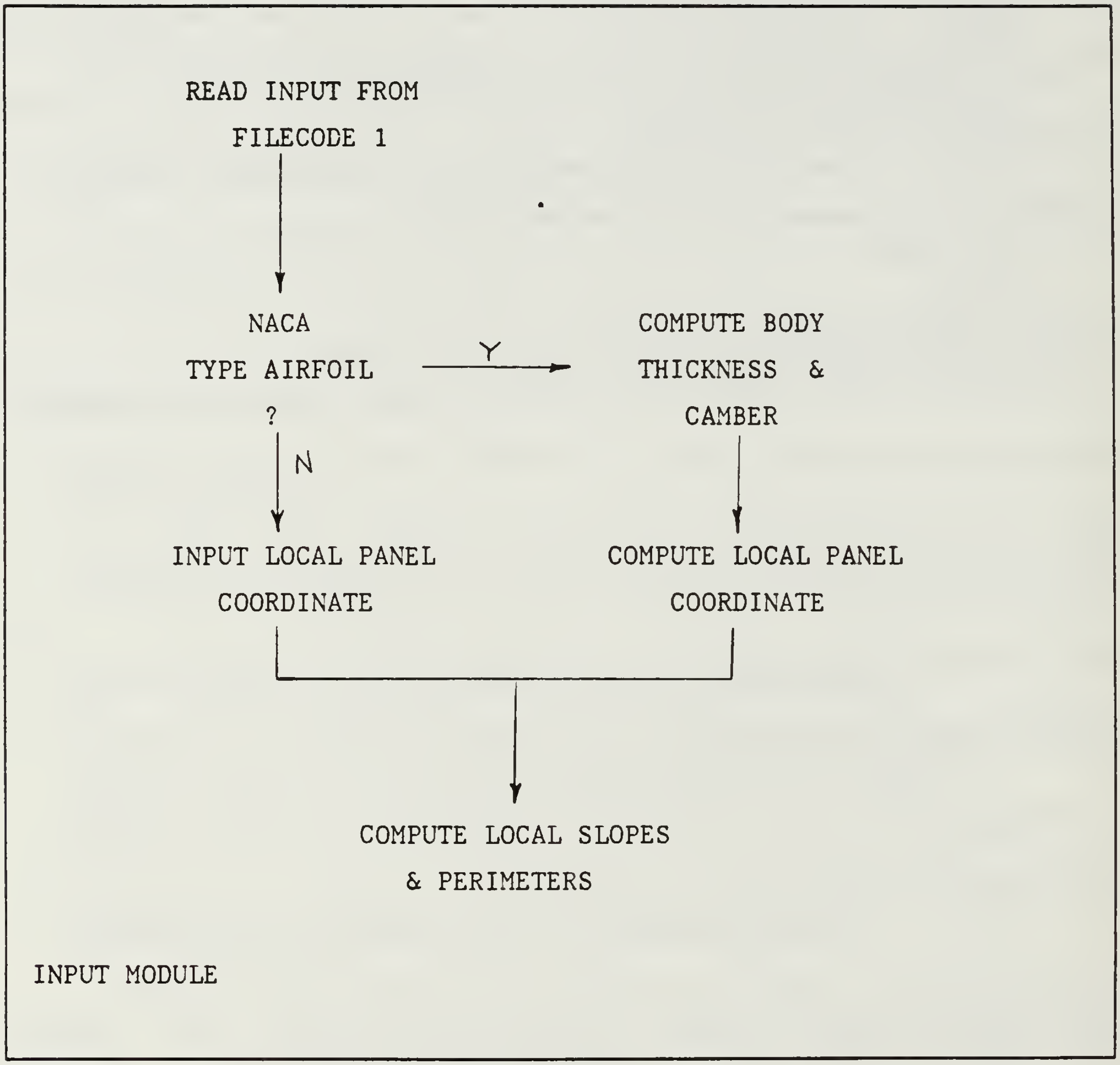


Figure 6. Flow Chart for USPOTF2 Computer Code.

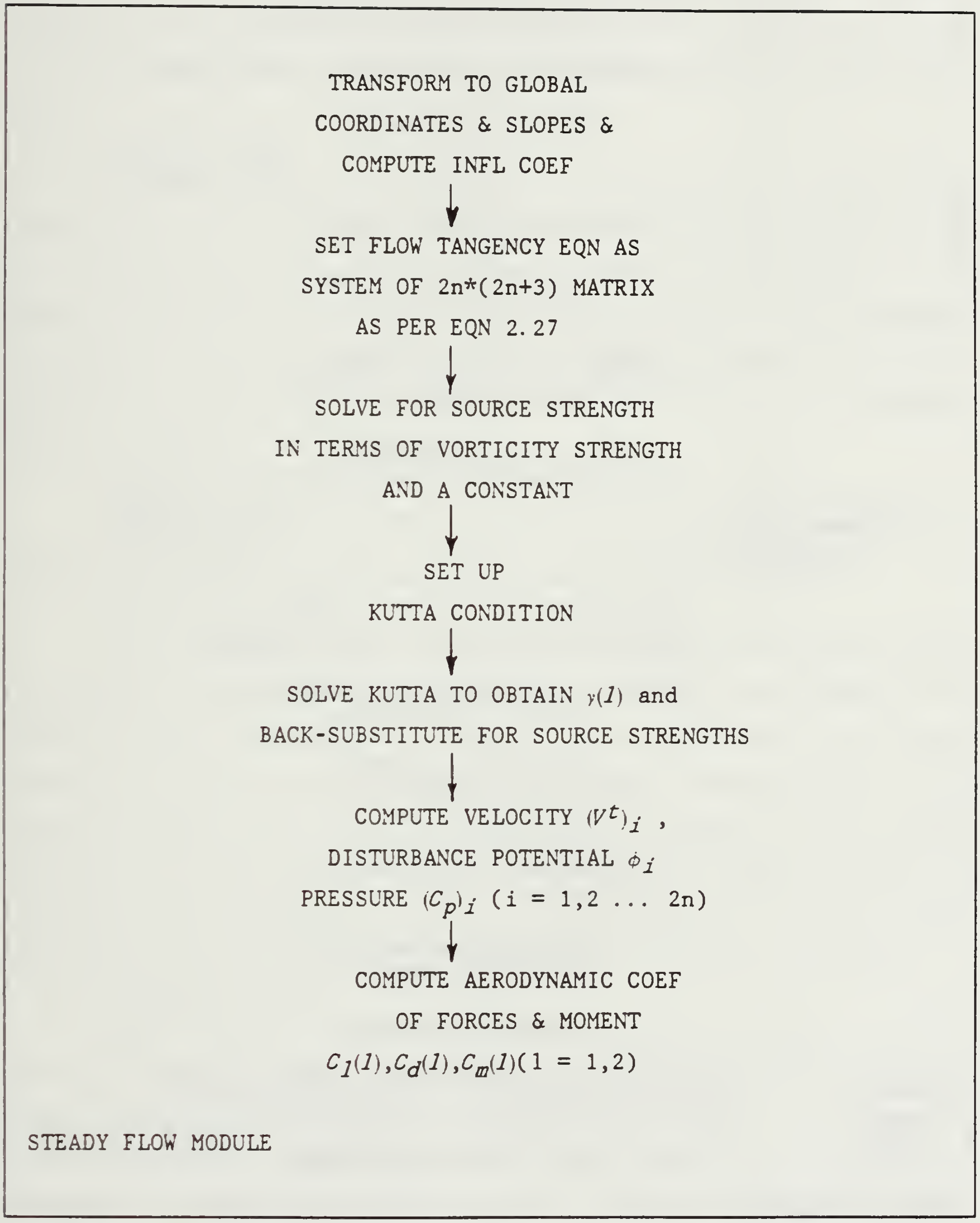

TRANSFORM TO GLOBAL
COORDINATES & SLOPES &
COMPUTE INFL COEF
SET FLOW TANGENCY EQN AS
SYSTEM OF 2n*(2n+3) MATRIX
AS PER EQN 2.27
SOLVE FOR SOURCE STRENGTH
IN TERMS OF VORTICITY STRENGTH
AND A CONSTANT
SET UP
KUTTA CONDITION
SOLVE KUTTA TO OBTAIN $\gamma(1)$ and
BACK-SUBSTITUTE FOR SOURCE STRENGTHS
COMPUTE VELOCITY $(V^t)_i$,
DISTURBANCE POTENTIAL ϕ_i
PRESSURE $(C_p)_i$ (i = 1,2 ... 2n)
COMPUTE AERODYNAMIC COEF
OF FORCES & MOMENT
$C_l(l), C_d(l), C_m(l) (l = 1,2)$
STEADY FLOW MODULE

Figure 6 (Cont'd)

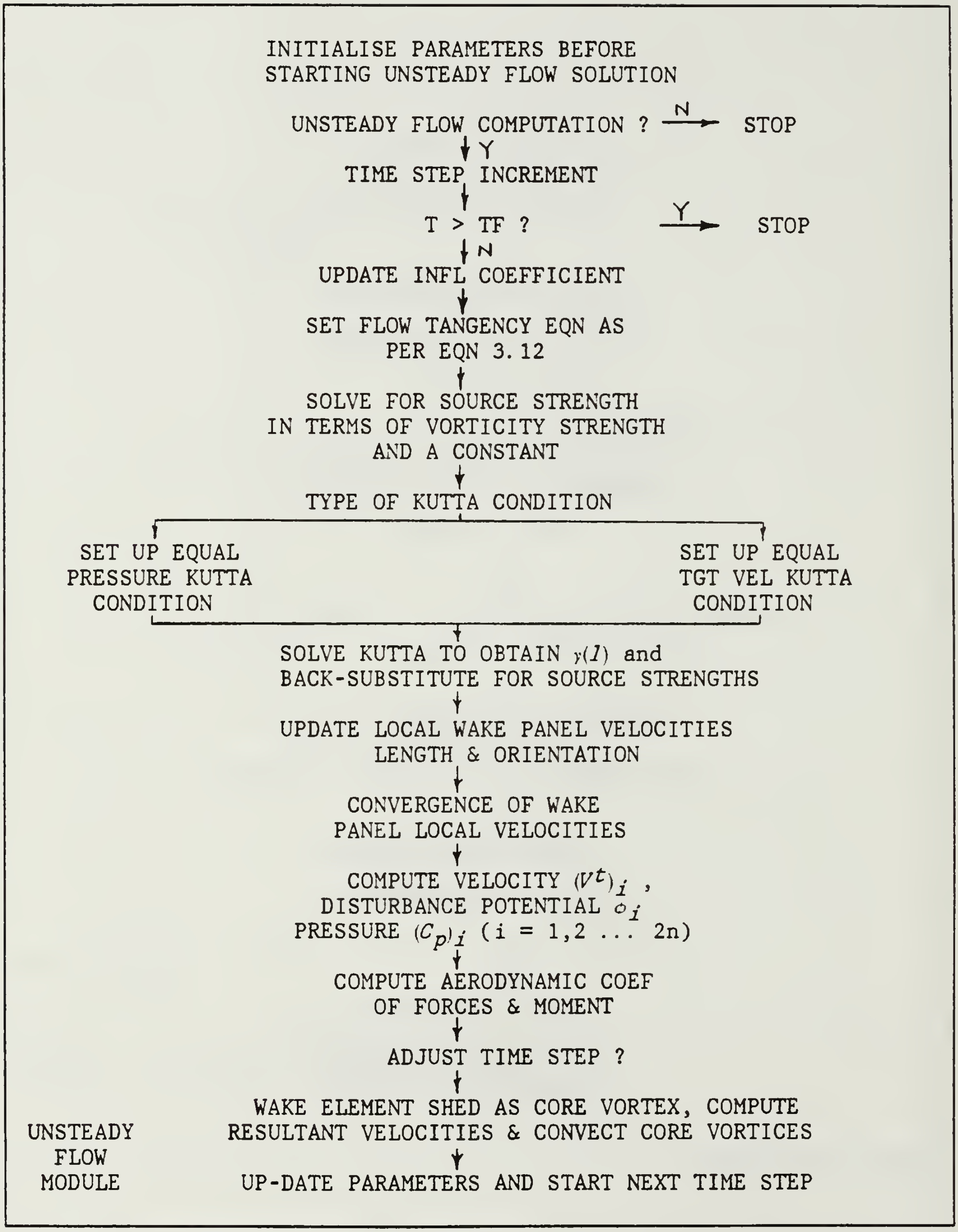

INITIALISE PARAMETERS BEFORE STARTING UNSTEADY FLOW SOLUTION
UNSTEADY FLOW COMPUTATION ?
N
STOP
Y
TIME STEP INCREMENT
T > TF ?
Y
STOP
N
UPDATE INFL COEFFICIENT
SET FLOW TANGENCY EQN AS PER EQN 3.12
SOLVE FOR SOURCE STRENGTH IN TERMS OF VORTICITY STRENGTH AND A CONSTANT
TYPE OF KUTTA CONDITION
SET UP EQUAL PRESSURE KUTTA CONDITION
SET UP EQUAL TGT VEL KUTTA CONDITION
SOLVE KUTTA TO OBTAIN $\gamma(1)$ and BACK-SUBSTITUTE FOR SOURCE STRENGTHS
UPDATE LOCAL WAKE PANEL VELOCITIES LENGTH & ORIENTATION
CONVERGENCE OF WAKE PANEL LOCAL VELOCITIES
COMPUTE VELOCITY $(V^t)_i$, DISTURBANCE POTENTIAL ϕ_i PRESSURE $(C_p)_i$ (i = 1,2 ... 2n)
COMPUTE AERODYNAMIC COEF OF FORCES & MOMENT
ADJUST TIME STEP ?
WAKE ELEMENT SHED AS CORE VORTEX, COMPUTE RESULTANT VELOCITIES & CONVECT CORE VORTICES
UP-DATE PARAMETERS AND START NEXT TIME STEP
UNSTEADY FLOW MODULE

Figure 6 (Cont'd)

B. DESCRIPTION OF SUBROUTINES

1. Subroutine BODY

This subroutine is called for the purpose of obtaining the local (x,y) coordinates of either a NACA XXXX or 230XX type airfoil. It is called by subroutine SETUP and it in turn calls subroutine NACA45 to obtain the airfoil thickness and camber distributions.

2. Subroutine COEF

This subroutine was originally intended for the unsteady flow calculation for the single airfoil case. It is now modified to include the steady flow calculations. Its purpose is to set up the flow tangency matrix as in equation 2.27 for the steady flow and equations 3.13 through 3.16 for the unsteady flow case. These matrices are necessarily set up in this way so that the source strengths can be solved in terms of the vorticity strengths and a constant by subroutine GAUSS as a linear system with three right hand sides. It is called by the MAIN program.

3. Subroutine COFISH (deleted for the two airfoil case)

This subroutine was originally set up to serve the same function as subroutine COEF for the steady case. It is now deleted for computational efficiency.

4. Subroutine CORVOR

This subroutine is called for the purpose of obtaining the convective velocities for all the wake core vortices with respect to the frozen local frame of reference of the current time step in accordance with equations 3.35 and 3.36 where all the influence coefficients are now locally computed (previously done in subroutine INFL) to save storage requirements since these are only required in this subroutine. The local influence coefficients are obtained indirectly by first obtaining the global influence coefficients and transforming them viz equation 3.9. This subroutine is called by the main program nearing the end of the unsteady flow calculations before starting a new time step.

5. Subroutine FANDM

This subroutine is intended to calculate the overrall lift coefficient, drag coefficient and moment coefficient about the leading edge for the two airfoils. In order to preserve the option of obtaining the x and y forces with respect to the respective local frames of reference, the original method rather than equations 2.34 through 2.36 is implemented. This subroutine is called by the MAIN program in both the steady and unsteady flow computations immediately after computing the pressure coefficient.

6. Subroutine GAUSS

This subroutine is the standard linear system solver that employs the well-known Gaussian elimination with partial pivoting and operates simultaneously on a user specified number of right-hand-sides. It is called by the MAIN program in both the steady and unsteady flow calculations. In order to use GAUSS, the coefficients of the augmented matrix must be set up so that GAUSS will return the solutions replacing the corresponding columns of the augmented matrix that were initially occupied by the right-hand-sides. The coefficient set-ups are done by subroutine COEF for both steady and unsteady flow problems.

7. Subroutine INDATA

This subroutine is intended to read in the first three to five cards of the input data depending on whether IFLAG = 0. The first three cards contain some description of the airfoil type, problem definition, IFLAG information as well as the number of lower and upper panels. If IFLAG = 0, it will treat the airfoils as NACA type airfoils and will proceed to read the NACA number and to calculate the thickness parameters that will be required by subroutine NACA45. This is the first subroutine called by the MAIN program.

8. Subroutine INFL

This subroutine generates most of the influence coefficients that are needed and shared by the different subroutines. It has been modified to include the steady flow case for the purpose of reducing repeated computations. It utilises the known relative geometrical parameters of the singularities to carry out computation based on equations 2.18 through 2.21 for the steady flow calculations and including 3.9 through 3.11 for the unsteady flow case. The MAIN program calls this subroutine in every iteration cycle of each time step so that the time dependent influence coefficients can be updated as and when neccessary. Time independent coefficients are computed once in the entire flow solutions[14].There are also time dependent influence coefficients that are independent of the iterative cycle to obtain the wake panel orientation[15]. Those influence coefficients involving the wake core vortices are also independent of the iteration cycle and are also updated once in each time step; but the process of the update is more complicated. This is due to the process that the wake need first to be convected with respect to the previous time step frozen frame of reference, transformed to the present frame of reference and finally transformed to the global frame of reference by subroutine NEWPOS. Finally the influence coefficients involving the wake panels are calculated as frequently as the number of iterations take to find a converged solution.

---

[14] These are: on the airfoil panel by the panels on the same airfoil.

[15] These are: on the airfoil panel by the panels on the other airfoil.

9. Subroutine KUTTA

This subroutine is intended to solve the Kutta equation. It has been modified to include the steady flow solution. After the source strengths have been determined by subroutines COEF and GAUSS in terms of the vorticity strengths and a constant, this subroutine invokes the Kutta condition as in equations 2.25, 2.26 for the steady flow case and 3.33, 3.34 for the unsteady flow case. The two linear equations[16] are then solved again with Gaussian elimination to obtain the vorticity distributions. For the unsteady case, we add in the additional requirement of finding the product of the tangential velocities at the upper and lower trailing edges. These are demanded to be negative as there are basically two possible solutions to the Kutta equations due to its original quadratic nature.

10. Subroutine NACA45

This subroutine is intended to calculate the camber and thickness distribution of the NACA 4-digit and the NACA 5-digit airfoils of type 230XX which share common thickness distributions with the 4-digit airfoil having the same thickness to chord ratio. This subroutine is called by subroutine BODY which is in turn called by subroutine SETUP and in turn called by the MAIN program.

11. Subroutine NEWPOS

This is a new subroutine introduced as a result of setting up the five frames of reference. Most of the coordinates computation is done with respect to the respective local frame of reference. Its purpose then is to transform all coordinates in the respective local frames of reference to the global frame of reference[17]. This is necessary for the computation of the influence coefficients as well as in the convection of the wake core vortices. It facilitates the simple requirements of defining the airfoil once only with respect to its local frame of reference. Orientation and displacement of the airfoil in the two-dimensional plane is henceforth calculated through this subroutine. This subroutine is called from the MAIN program in several locations immediately after the local coordinates are computed.

---

[16] For the unsteady case, the original two equations are non-linear and were subsequently linearised in the discussion in Chapter 3.

[17] It has a secondary function to obtain the slopes for the airfoil panels and the wake element panels.

12. Subroutine PRESS

This subroutine calculates the pressure distribution over the airfoil panels after the iterative solution for the unsteady flow problem has successfully met the convergence criterion. It first computes the tangential velocities at all panel control points using equation 3.40 (with $p = \frac{P+1}{2}$), then performs the disturbance potential evaluation at the current time step according to equations 3.41 through 3.43. Together with the disturbance potential data obtained from the previous time step, it calculates the pressure distribution using equation 3.37.

13. Subroutine SETUP

This subroutine sets up the local panel nodal coordinates for MAIN program by reading the fourth through seventh data sets of the input file if IFLAG = 1 is set. It skips the data reading if IFLAG = 0 and proceeds to set up the node distribution and calls subroutine BODY to calculate the airfoil local coordinates. The node distribution adopts a cosine formula in order to have closely packed panels toward the leading and trailing edges for improvements in solution accuracy. Regardless of how the nodal coordinates are obtained , subroutine SETUP determines the local panel slopes and airfoil perimeter length.

14. Subroutine TEWAK

This subroutine is intended to calculate the resultant velocity components at the mid-point of the shed vorticity panel with respect to the current frozen frame of reference. These velocity components are necessary to ensure that the correct shed vorticity panels' length and orientation are established. This is the governing criterion for the iterative solution scheme of the unsteady flow case. This subroutine is called by the MAIN program at every iteration cycle of each time step for the unsteady flow calculation.

15. Subroutine VELDIS

This subroutine essentially performs the same function as subroutine PRESS for the steady flow case. It is thus redundant and could be deleted with some modification work necessary for subroutine PRESS. This work has been implemented by Krainer for the single airfoil case. This subroutine is called by the MAIN program in the steady flow computation.

C. INPUT DATA FOR PROGRAM USPOTF2

The Input data is similar to that for the single airfoil case. Program USPOTF2 reads its input data from filecode 1. An example of an input data file is attached in Appendix B for the case when the airfoil nodal coordinates are input by the user. Computer generated airfoil coordinates are another option that can be selected if the airfoil chosen belongs to the family of NACA 4-digit or 5-digit airfoils of type 230XX. To do this, the IFLAG parameter is set to zero in the first item of the 4th set of data card and replace the 5th and 6th set of cards by two cards containing the particular airfoil NACA number for the two airfoils using format (I5). Figure 7 contains an itemised description of the sequential input variables.

D. OUTPUT DATA FROM PROGRAM USPOTF2

The Output data is similar to that for the single airfoil case. Appendix C contains a sample output data generated by using the input data set from Appendix B. Due to the repetitive nature of the output as a function of time, only selective time set data are shown. The output data file begins with writing out what the program has read from the data file followed by the computed nodal coordinates only if they are generated by the program: otherwise it proceeds to write the airfoil computed perimeter length. The next set of output data are the steady flow solution parameters of distributed source strengths, vorticity strengths, pressure-velocity distribution, force-moment coefficient, potential at the control nodal points and the potential at the leading edge. In addition some output is given to allow an assessment of the accuracy of the flow solution. This consists of the normal component of the velocities at the panel control points and the integral of the disturbance tangential velocity along the airfoil contour. If only the steady flow solution is required, the output terminates here.

For unsteady flow, in addition to the above, the unsteady flow parameters are also printed. This includes, at every time step, the iterative printout for the convergence of the length and orientation of the wake element panel, the unsteady solution parameters on the airfoil similar to that of the steady flow parameters as well as the trailing wake core vortices data. Again the same checking mechanisms are inserted on the airfoil panels. A detailed explanation on the output variable names is listed in Figure 8. All output data are non-dimensional quantities.

Data Set #1	Format (I5) - 1 data card.
ITITLE	- Number of title cards to be used in Data Set #2.
Data Set #2	Format (20A4) - ITITLE data cards.
TITLE	- Headings printed on output for case run identification.
Data Set #3	Format (I5,2F10.6) - 1 data card.
NAIRFO	- Number of airfoil, in this case = 2.
XSHIFT	- Relative X distance of the 2 airfoil's pivot position with respect to the airfoil global coordinate system.
YSHIFT	- Relative Y distance of the 2 airfoil's pivot position with respect to the airfoil global coordinate system.
Data Set #4	Format (3I5) - 1 data card.
IFLAG	- 0 if airfoil is NACA XXXX or 230XX. - 1 otherwise.
NLOWER	- Number of panels used on both airfoil lower surface.
NUPPER	- Number of panels used on both airfoil upper surface.
Data Set #5	Format (6F10.6) if IFLAG = 1 - variable data cards.
x(I),y(I)	- local non-dimensionalised x-nodal followed by y-nodal coordinates for airfoil 1. Total of Nlower+Nupper+1 nodal points divided into 6 points per data card.
Data Set #6	Format (6F10.6) - variable data cards
x(I),y(I)	- local non-dimensionalised x-nodal followed by y-nodal coordinates for airfoil 2. Total of Nlower+Nupper+1 nodal points divided into 6 points per data card.

Figure 7. List of Input Variables.

Data Set #7	Format (6F10.6,I3) - 1 data card
ALPI(1)	- Initial angle of attack (AOA) for airfoil 1 in degree.
ALPI(2)	- Initial angle of attack (AOA) for airfoil 2 in degree.
DALP	- Absolute change in AOA in degree for non oscillatory motions. - Maximum amplitude of AOA change in degree for rotational harmomic motions.
TCON	- Non-dimensional rise time ($V_\infty t/c$) of AOA for motion involving modified-ramp change in AOA.
FREQ	- Non-dimensional oscillation ($\omega c/V_\infty$) for harmonic motions.
PIVOT	- The length from the leading edge to the pivot point for the local system.
IPHASE	- Flag for in-phase and out-of-phase motion. - 0 out-of-phase motion. - 1 in-phase motion.
Data Set #8	Format (8F10.6) - 1 data card.
UGUST	- Magnitude of non-dimensional gust velocity along global x-direction.
VGUST	- Magnitude of non-dimensional gust velocity along global y-direction.
DELHX(1)	- Non-dimensional translational chordwise amplitude for airfoil 1
DELHX(2)	- Non-dimensional translational chordwise amplitude for airfoil 2

Figure 7 (cont'd)

Data Set #8	(Cont'd)
DELHY(1)	- Non-dimensional translational transverse amplitude for airfoil 1
DELHY(2)	- Non-dimensional translational transverse amplitude for airfoil 2
PHASE(1)	- Phase angle in degree between the chordwise and transverse translational oscillation with the latter as reference for the first airfoil.
PHASE(2)	- Phase angle in degree between the chordwise and transverse translational oscillation with the latter as reference for the second airfoil.
Data Set #9	Format (5F10.6,I5) - 1 data card.
TF	- Final non-dimensional time to terminate unsteady flow solution.
DTS	- Starting time step for non-osc. motions if TADJ = 0.0 - No. of computational steps per cycle for harmonic motion. - Baseline time step size for all motions if TADJ $\neq$ 0.0
TOL	- Tolerance criterion for checking the convergence between successive iterations of $(U_w)_k$ and $(V_w)_k$.
TADJ	- Factor by which DTS will be adjusted.
SCLA	- Steady lift coefficient for the single airfoil at the specified AOA
NGIES	- Option for changing the unsteady Kutta condition to satisfy the tangential velocity as per Geising's case. - 0 equal pressure at the trailing edge panels. - 1 equal tgt velocities at the trailing edge panels.

Figure 7 (cont'd)

TK - Time step t_k.
TKM1 - Time step t_{k-1}.
ALPHA(L) - Angle of attack of airfoil L at time t_k.
OMEGA(L) - Rotational velocity (positive counter clockwise) at time t_k for airfoil L
U(L) - Chordwise translational velocity (positive forward) at time t_k for airfoil L
V(L) - Transverse translational velocity (positive downward) at time t_k for airfoil L
NITR - Iteration number.
VXW(L) - Iterative solution of $(U_w)_k$ of airfoil L
VYW(L) - Iterative solution of $(V_w)_k$ of airfoil L
WAKE(L) - Iterative solution of shed vorticity panel length Δ_k of airfoil L
THETA(L) - Iterative solution of shed vorticity panel orientation Θ_k of airfoil L
GAMK(L) - Iterative solution of the strength of the current vorticity distribution of airfoil L
J - Panel number.
XI(J) - local x-coordinate of the midpoint of j^{th} panel.
YI(J) - local y-coordinate of the midpoint of j^{th} panel.
X(J) - global X-coordinate of the midpoint of j^{th} panel.
Y(J) - global Y-coordinate of the midpoint of j^{th} panel.
Q(J) - Strength of source distribution on the j^{th} panel.
CP(J) - Pressure coefficient at the midpoint of the j^{th} panel.
V(J) - Total tangential velocity at the midpoint of the j^{th} panel with respect to the moving local system.

Figure 8. List of Output Variables.

VN(J) - Normal velocity at the midpoint of the j^{th} panel with respect to the moving local system.

PHIK(J) - Potential at the mid-point of the j^{th} panel at the current time step.

PHI(J) - Potential at the mid-point of the j^{th} panel at previous time step.

INTGAMMA - Integral of the disturbance velocity around the airfoil.

CD(L) - Drag coefficient of airfoil L.

CL(L) - Lift coefficient of airfoil L.

CM(L) - Pitching moment coefficient about leading edge of airfoil L.

M - Trailing wake core vortex number.

X1I(M) - X-coordinate of the center of the m^{th} core vortex of airfoil 1 with respect to local system.

Y1I(M) - Y-coordinate of the center of the m^{th} core vortex of airfoil 1 with respect to local system.

X1(M) - X-coordinate of the center of the m^{th} core vortex of airfoil 1 with respect to global system.

Y1(M) - Y-coordinate of the center of the m^{th} core vortex of airfoil 1 with respect to global system.

X2I(M) - X-coordinate of the center of the m^{th} core vortex of airfoil 2 with respect to local system.

Y2I(M) - Y-coordinate of the center of the m^{th} core vortex of airfoil 2 with respect to local system.

X2(M) - X-coordinate of the center of the m^{th} core vortex of airfoil 2 with respect to global system.

Y2(M) - Y-coordinate of the center of the m^{th} core vortex of airfoil 2 with respect to global system.

CIRC(M,L)- Circulation strength of the m^{th} core vortex of airfoil L

Figure 8 (Cont'd)

V. RESULTS AND DISCUSSION OF CASE-RUNS

USPOTF2 is primarily written as a follow-up to U2DIIF for the single airfoil. All the case-runs with the exception of the gust case will be presented. The approach in the gust case is not consistent with the requirements for an irrotational flow field and will not be treated in this report. The step change in angle of attack (AOA) will be compared with Giesing's for the same airfoil set at the final AOA, undergoing an impulsive start from rest. As there exist no comparison data for the other sub-cases, the results will thus not be as extensive as the step input. They are documented here for the sole purpose of illustrating the capability of the code.

A. STEP CHANGE IN ANGLE OF ATTACK

1. Case Run Definition

Consider a first case of two airfoils initially at zero AOA to the free stream V_∞ which undergo an out-of-phase step change in AOA (α_{step}) at time t_0. Consider a second case of two airfoils at rest, set initially at an AOA of α_{step} out-of-phase with one another, given an impulsive start to V_∞. The above two cases are equivalent within the thin airfoil approximation. Case 1 is computed by USPOTF2 and case 2 is obtained by Giesing's computational analysis.

2. Differences between USPOTF2 and Giesing's code

While both codes use an extension of the PANEL method to solve for the unsteady potential flow solution for two airfoils, perfect correlation is not possible for several reasons. The reader is referred to References [5] and [7] for a detailed description of Giesing's approach. Some basic differences are:

- USPOTF2 models the wake vortex sheet by a wake element and a series of point vortices shed through the wake element. This follows the approach of Basu and Hancock with the necessary assumptions on the wake element characteristics. Giesing treated the wake vortex sheet to comprise of a distributed line vortex which is convected at the local fluid velocity at the vortex location assuming that the small portion of the wake being shed does not contribute to the convection of the wake.
- The time step increment in USPOTF2 is a parameter that affects the overall results of the unsteady flow in that by having small time step, the point vortices being shed will be greater in number but weaker in strength, while for bigger time step, the point vortices become fewer but have effectively stronger strengths. The time step increment in Giesing's code is important in that the assumption that the small

portion of the wake being shed does not contribute to the convection of the wake is only exactly true in the limit when the time step becomes zero.

- Giesing uses the Adam's formula of varying degree[18] to convect the wake vortex sheet while USPOTF2 uses only a predictor algorithm.
- The treatments of the circulation $\Gamma(l)$ are different for both codes. USPOTF2 extended the influence coefficient concept to the unsteady flow regime and solved for the circulation using the Kutta condition of equations 3.33 and 3.34. All calculations were done in the moving frame in the presence of the unsteady wake formations. Giesing considered the circulation to be a combination of the quasi-steady circulation and the circulation due to the vortex wakes. From Reference [5]

  $$\Gamma(l) = \Gamma_Q(l) + \Gamma_s(l) \tag{5.1}$$

  where $\Gamma_Q(l)$ is the circulation required to satisfy the Kutta condition on body (l) as it moves through the fluid when it is assumed that the body does not shed any vorticity and $\Gamma_s(l)$ is that circulation required to satisfy the Kutta condition on body (l) as it travels through the flow field generated by the vortex wakes shed by the two bodies and the flow field generated by the circulatory flow about the other body.
- Giesing's Kutta condition reqires equal tangential velocities at the upper and lower surface panels at the trailing edges while the Kutta condition of USPOTF2 prescribes equal pressures. In order to have a meaningful comparison, USPOTF2 includes an option for equal tangential velocities.
- USPOTF2 requires the actual computation of the total velocity potential from a combination of an onset flow potential, disturbance potential and an assumption of a reference potential. Giesing uses the technique of the Douglas Potential Flow program which treats the potential as the combination of the quasi-steady potential and the potential due to the vortex wakes where these two terms are defined as before. The total velocity potential, in his case, need not be computed but only the time derivative of it which is then written in terms of previously known parameters.
- The number of panels and its distribution on the airfoils are different for both codes. This will be accentuated at the trailing edges and will lend itself to differences in the trailing edge panel distribution.

3. Results and Discussions

Figure 9 shows Giesing's results for the Von Mises 8.4 per-cent thick, symmetrical airfoil undergoing an impulsive start. This can be compared with Figure 10 which shows the result obtained with USPOTF2 for the same time step with NGIES set to one[19]. The results are not perfectly correlated but the quality and order of magnitude agreement are excellent. Figure 11 gives essentially the same result but with NGIES set to zero[20]. Surprisingly, the results obtained for both types of Kutta condition turn out to be quite similar. This is seen especially in the pressure coefficient plots when the two plots are put together as seen in Figure 12. The pressure coefficient agrees over 70 per-cent of chord length at the lower surface and over 90 per-cent of chord length at the upper surface with the greatest discrepancy at the trailing edge. Further comparison for smaller angles of attack would be of interest to see whether this similarity is generally true.

---

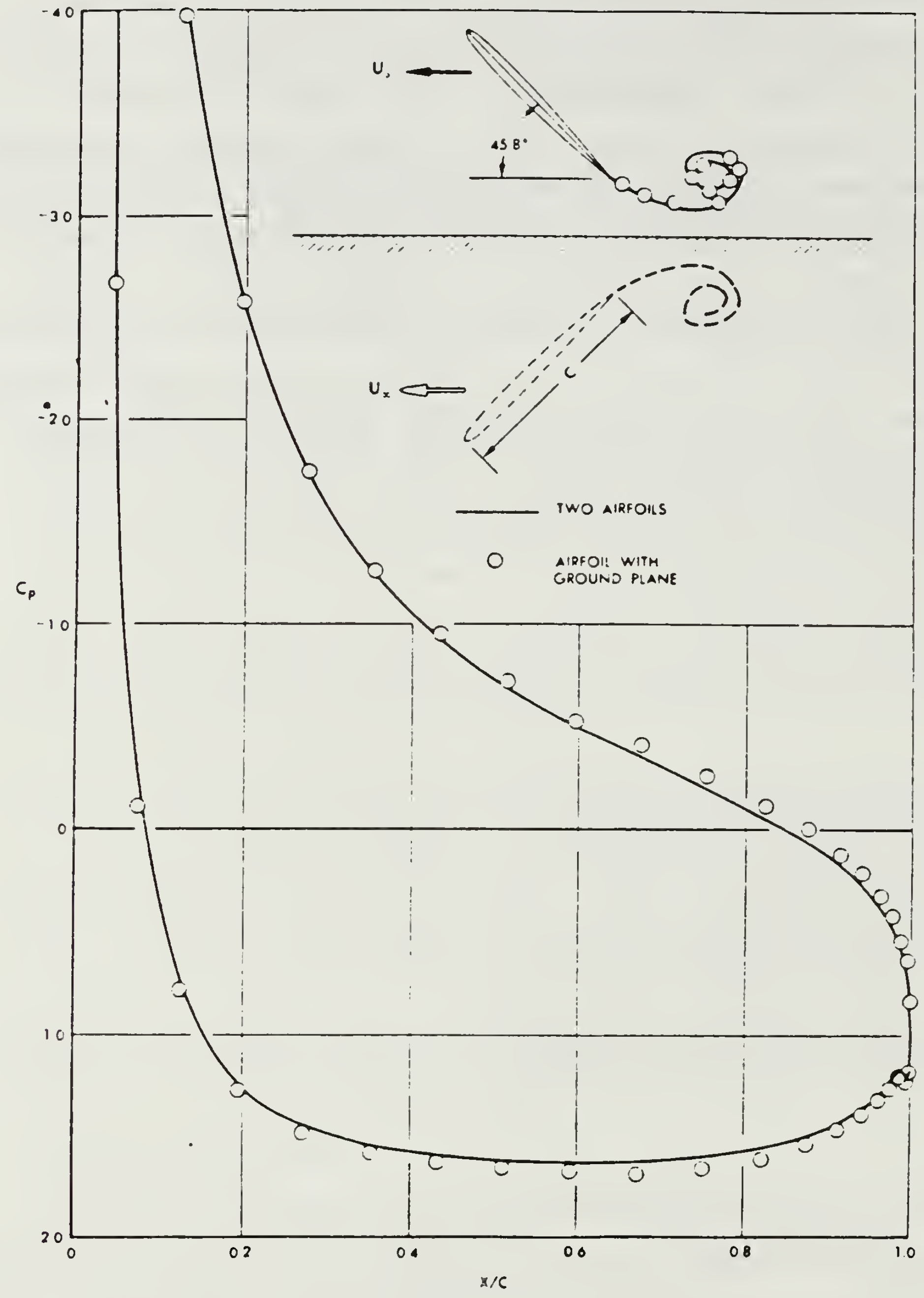
[18] This is essentially a predictor-corrector algorithm.

[19] This Kutta condition results in equal tangential velocities at the trailing edge panels of the airfoils.

Figure 13 compares the aerodynamic characteristics when the vertical distance YSHIFT is varied. Figure 14 gives the time variation for a larger time step of the normalised lift, moment coefficient and drag coefficient for the particular case of YSHIFT = 2.0.

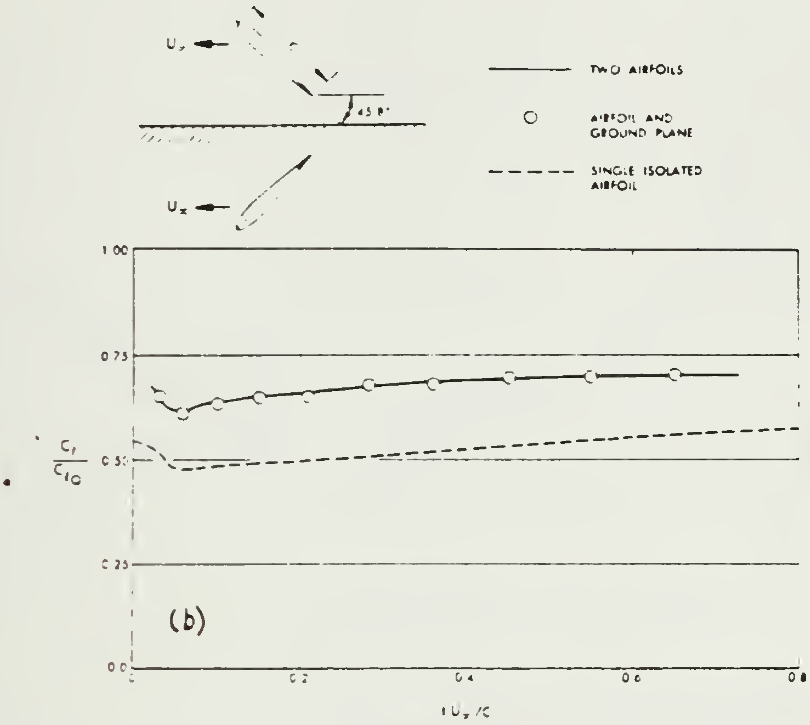
---

[20] This Kutta condition results in equal pressure coefficients at the trailing edge panels of the airfoils.



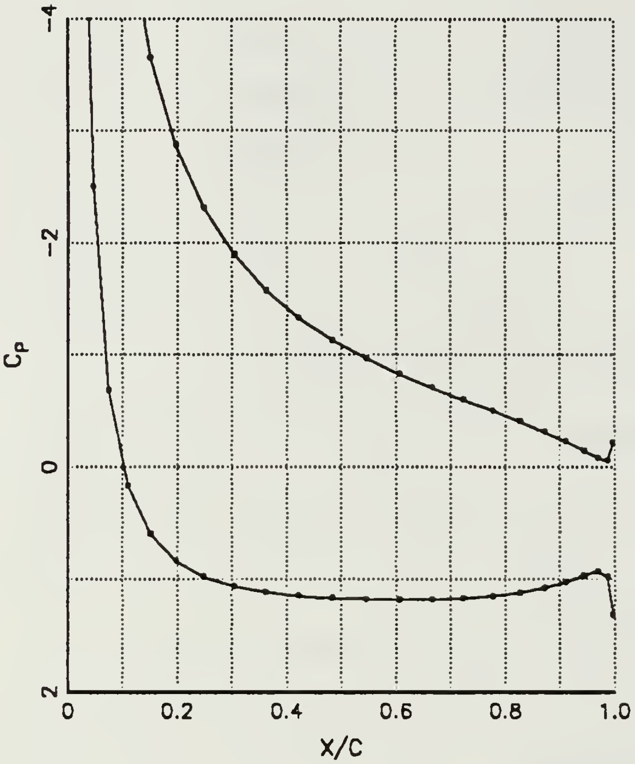
(a) Pressure Distribution and Vortex locations.

Figure 9. **Giesing's Calculated Results:** Impulsive Start for an 8.4 per-cent thick Von Mises airfoil set at $\alpha = 0.8$ radians for YSHIFT = 2.0 [reproduced with permission from Ref. 5].



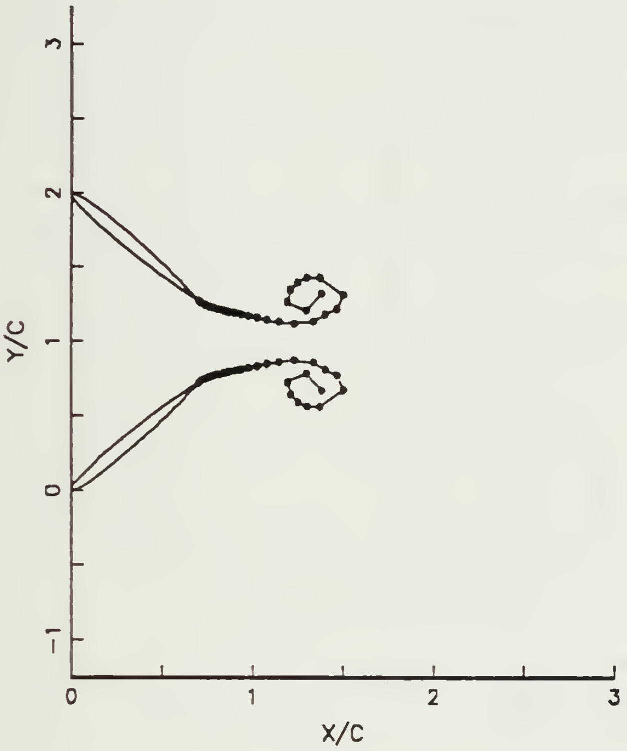
(b) Time History of the Lift Coefficients.

Figure 9 (Cont'd)



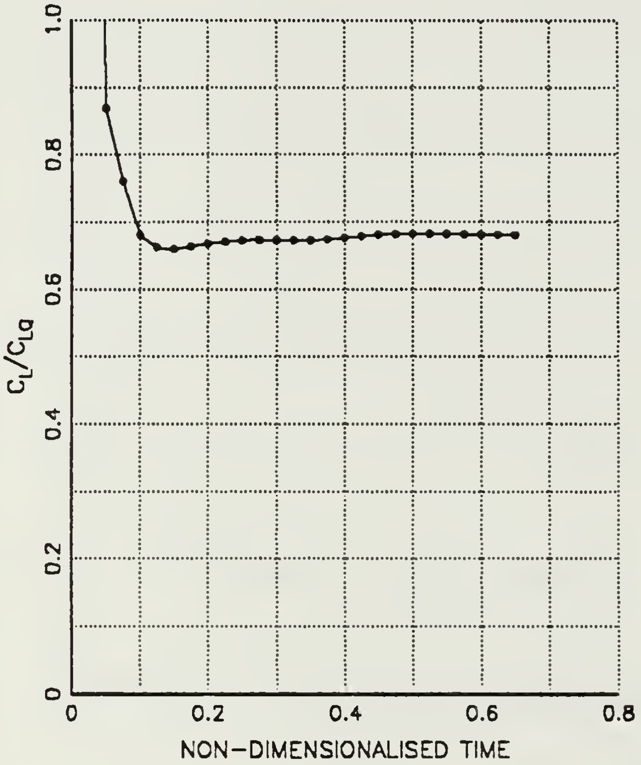
(a) Pressure Distribution at $tV_\infty/c = 0.65$

Figure 10. **USPOTF2 Results obtained with USPOTF2 for equal velocities at the trailing edge.:** Step change in AOA for an 8.4 per-cent thick Von Mises airfoils placed at 2 chord length vertical distance with initial AOA = 0.0 radians and final AOA = 0.8 radians pivoting at the leading edges.



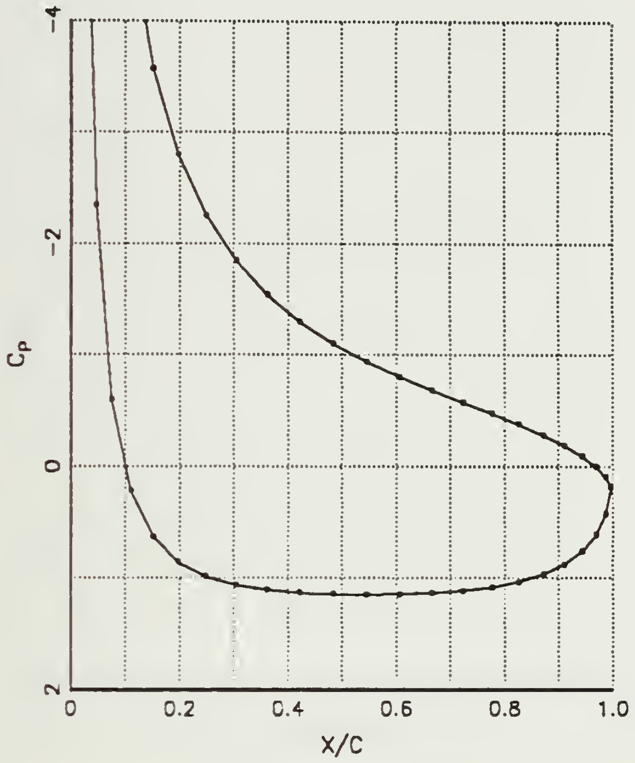
(b) Wake pattern at $tV_\infty/c = 0.65$

Figure 10 (Cont'd)



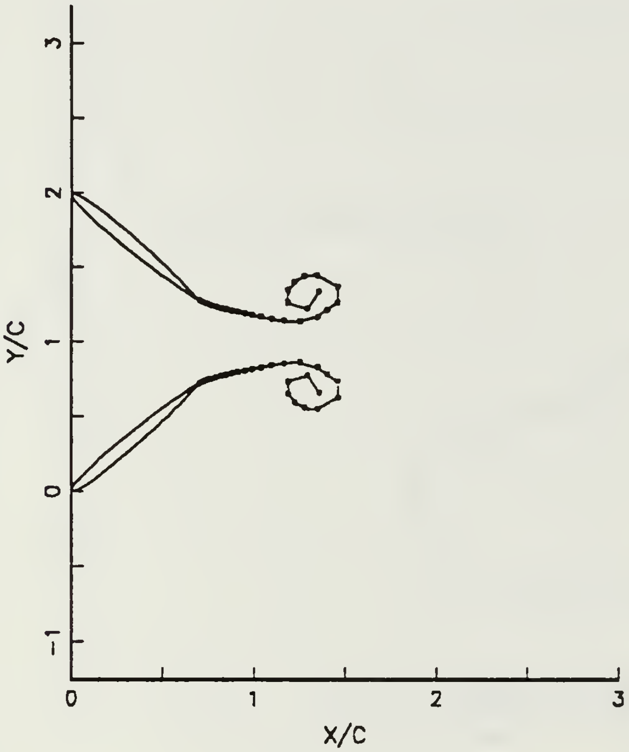
(c) Time History of the Lift Coefficients.

Figure 10 (Cont'd)



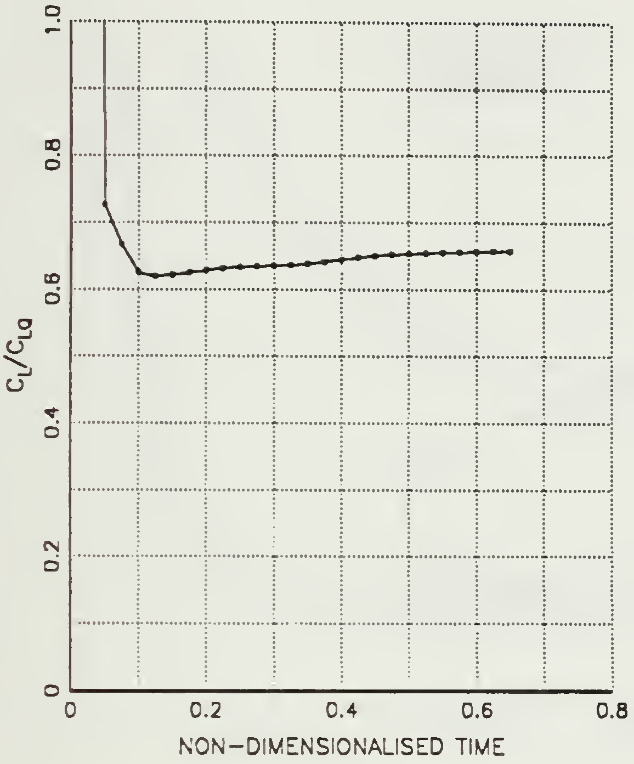
(a) Pressure Distribution at $tV_\infty/c = 0.65$

Figure 11. **USPOTF2 Results obtained with USPOTF2 for equal pressures at the trailing edge.:** Step change in AOA for an 8.4 per-cent thick Von Mises airfoils placed at 2 chord length vertical distance with initial AOA = 0.0 radians and final AOA = 0.8 radians pivoting at the leading edges.



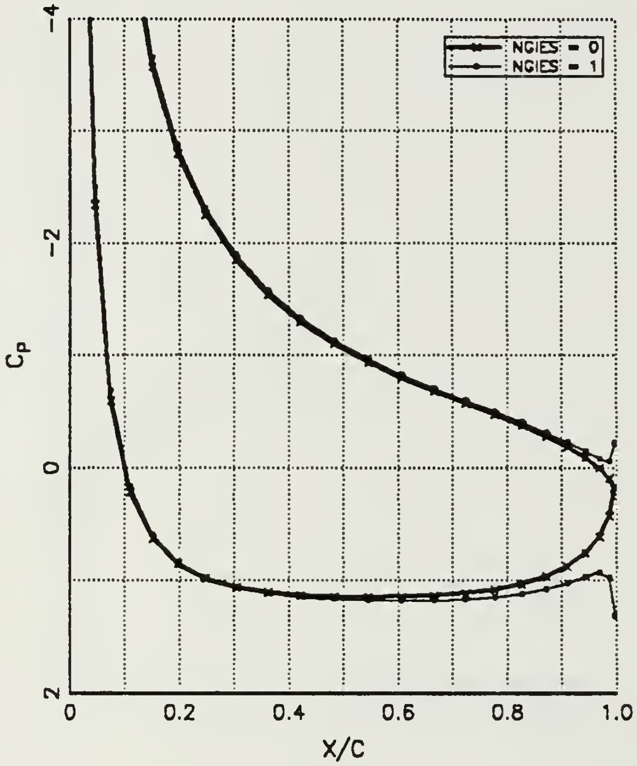
(b) Wake pattern at $tV_\infty/c = 0.65$

Figure 11 (Cont'd)



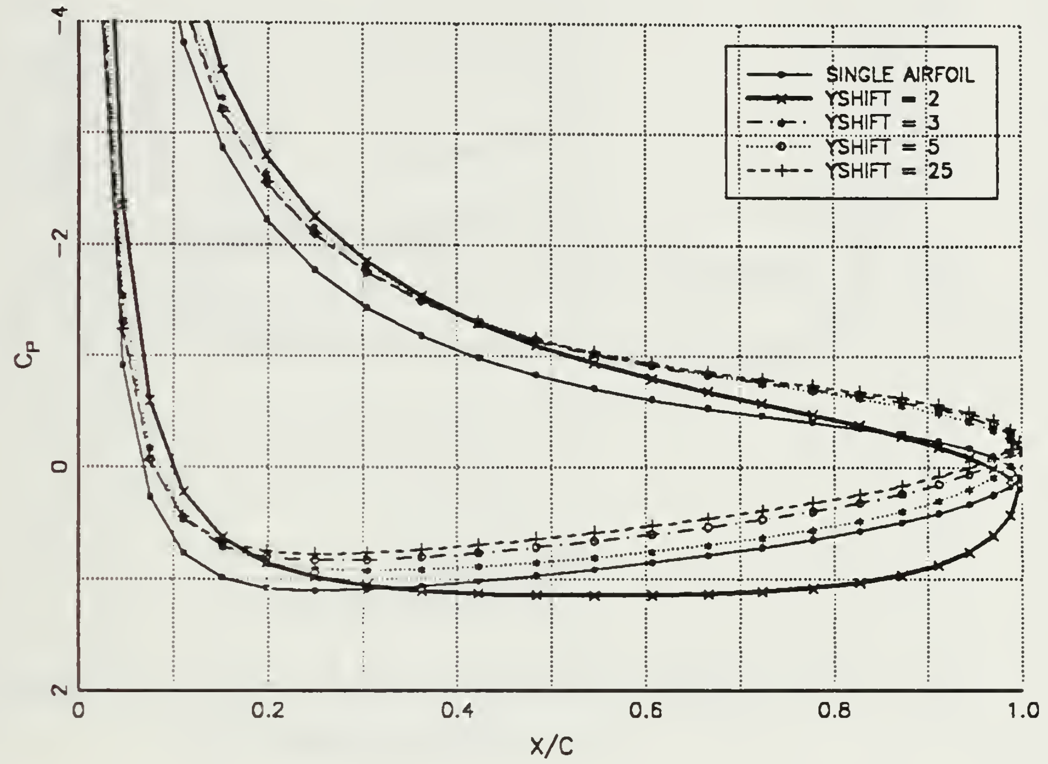
(c) Time History of the Lift Coefficients.

Figure 11 (Cont'd)



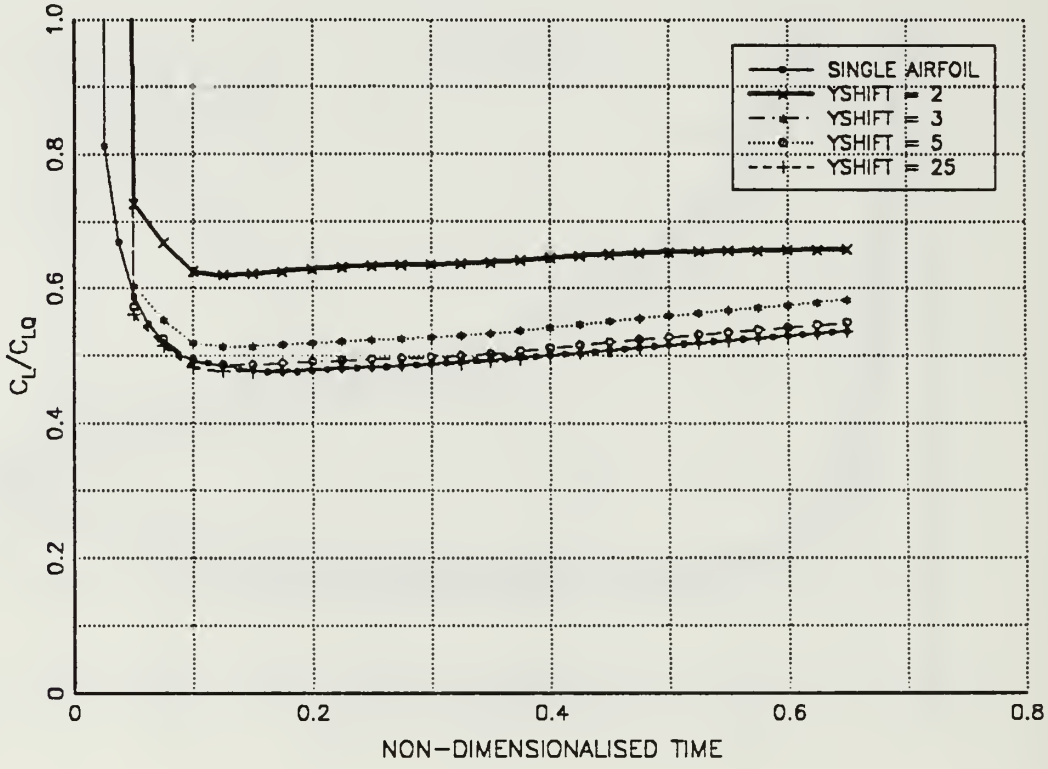
NGIES = 0 : Equal Pressure Coefficient at trailing edge panels.
NGIES = 1 : Equal Tangential Velocities at trailing edge panels

Figure 12. Pressure Distributions with different Kutta condition.



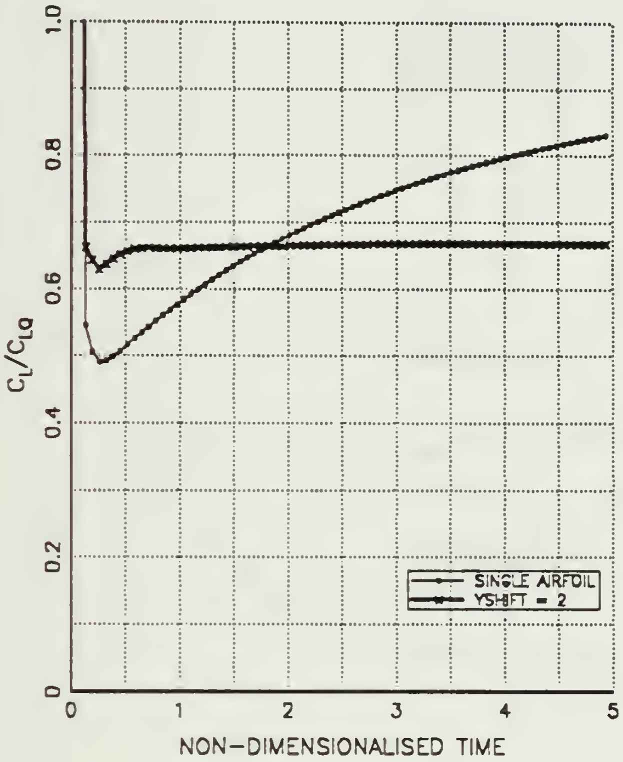
(a) Pressure Distribution at $tV_\infty/c = 0.65$

Figure 13. Pressure Distribution and Lift History as a function of the Vertical distance between the airfoils.



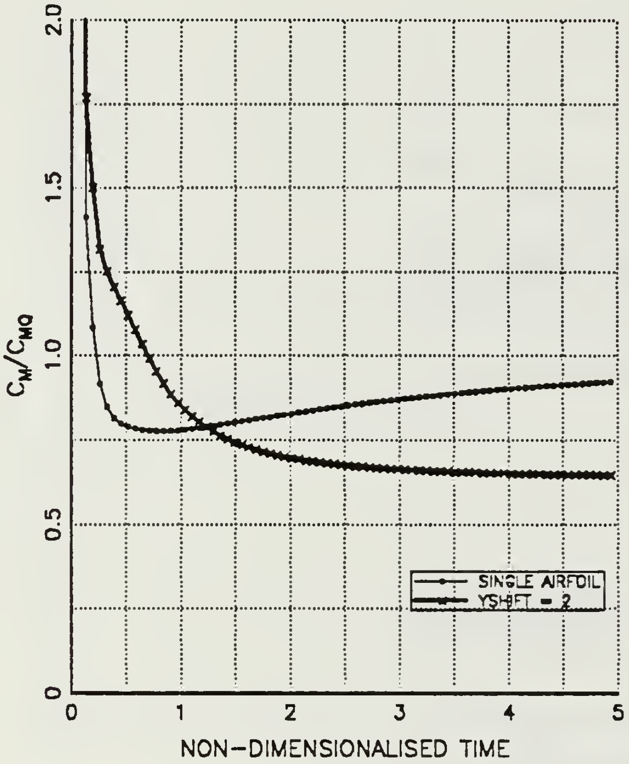
(b) Time History of the Lift Coefficients.

Figure 13 (Cont'd)



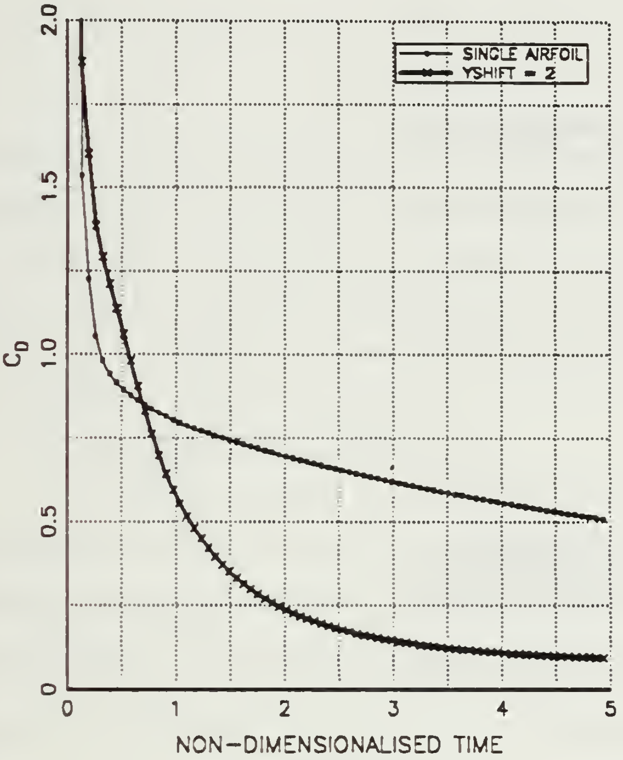
(a) Time History of the Lift Coefficients.

Figure 14. **Step Change in Angle Of Attack:** Step change in AOA for an 8.4 per-cent thick Von Mises airfoils placed at 2 chord length vertical distance with initial AOA = 0.0 radians and final AOA = 0.8 radians pivoting at the leading edges.



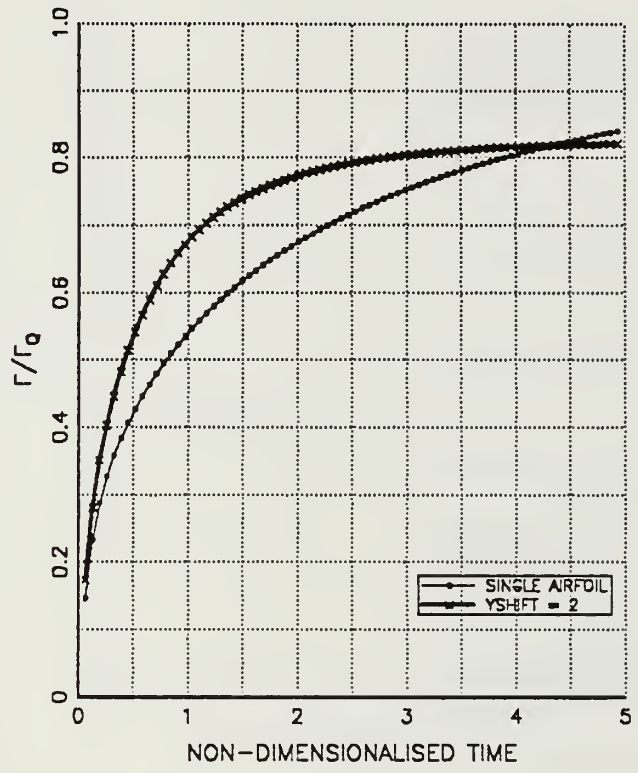
(b) Time History of the Moment Coefficients.

Figure 14 (Cont'd)



(c) Time History of the Drag Coefficients.

Figure 14 (Cont'd)



(d) Time History of the Circulation

Figure 14 (Cont'd)

B. OTHER SUB-CASES

The reader is referred to the results and disussions of Reference [1] for the comparison of the single airfoil case with existing codes in the phase and magnitude relationship of the aerodynamic coefficients to the forcing function as well as the wake convection. The present results show that the same trend is maintained as in the single airfoil case with no significant phase shift but with a general magnitude change due to an effective ground effect caused by the presence of the second airfoil.

1. Modified Ramp Change in Angle of Attack

As for the single airfoil, the modified ramp is defined mathematically as follows:

$$\alpha(t) = \begin{cases} 0 & t < 0 \\ \delta\alpha(3 - 2t/\tau)t^2/\tau^2 & 0 \leq t \leq \tau \\ \delta\alpha & t > \tau \end{cases} \tag{5.2}$$

where $\delta\alpha$ is the magnitude of the AOA change and τ is the rise time for the AOA to reach its final value. Figure 15 treats the case of the 2 airfoils undergoing an out-of-phase modified ramp change in the angle of attack of 0.1 radians with a time constant of 1.5. The plots show the effects on the aerodynamic coefficients due to the presence of the second airfoil. The corresponding aerodynamic coefficients of the single airfoil are plotted for comparison purposes.

2. Translational Harmonic Motion

The code is capable of computing the unsteady flow solution for any general translational motion described by a chordwise and a transverse component bearing a given phase relationship with the restriction that the 2 airfoils move only in-phase or out-of-phase. The translational harmonic motion is described by

$$h_y(t) = \delta h_y \, \mathrm{Sin}\, (\omega t)$$

$$h_x(t) = \delta h_x \, \mathrm{Sin}\, (\omega t + \lambda) \tag{5.3}$$

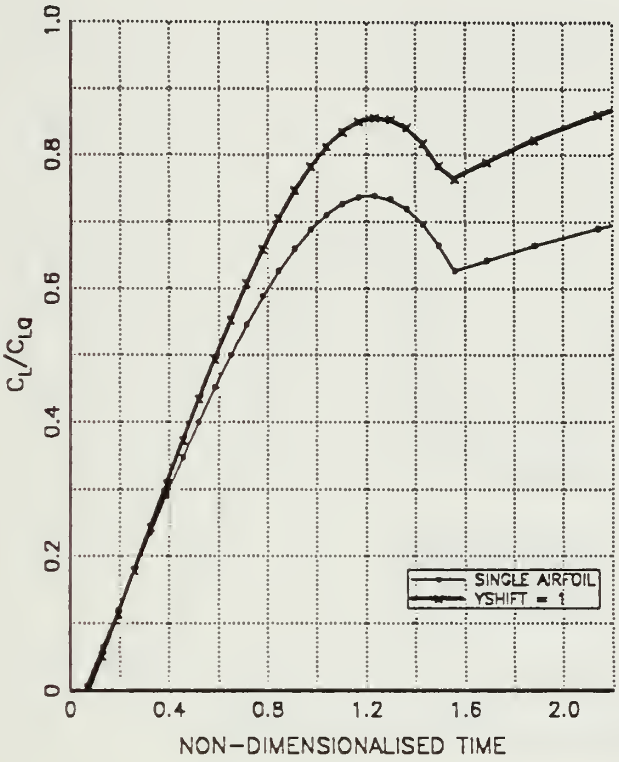
where ω is the oscillation frequency, λ is the phase angle between the chordwise and transverse oscillation and δh_x and δh_y are the magnitudes of chordwise and transverse oscillations respectively. The case-run considered in this section relates to a pure heaving or plunging motion. A NACA-0015 airfoil is chosen for the case-run. The airfoils are set at zero radian angles of attack and subsequently given an out-of- phase plunging oscillation at an amplitude of 0.018 chord length at a non-dimensionalised frequency of 1. Figure 16 shows the aerodynamic coefficients for the two airfoils and compares them with the single airfoil case where applicable. Note that the plots for the trailing wake uses different scales for the x and y axes.

3. Rotational Harmonic Motion

The treatment of the harmonic pitching motion is similar to the modified ramp case. As for the other sub-cases, the airfoils are restricted to in-phase and out-of phase motion. The case of the out-of-phase motion will be treated here. The harmonic pitching oscillation is described by:

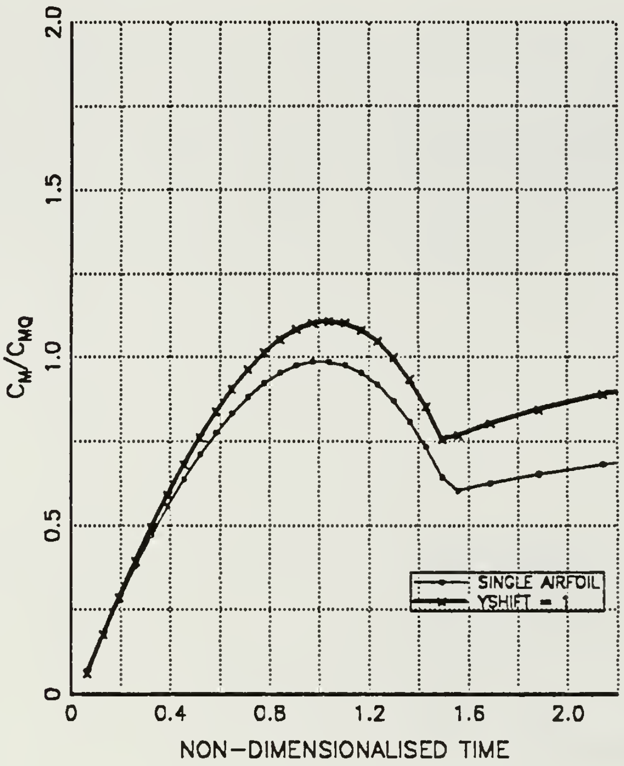
$$\alpha(t) = \delta\alpha \text{ Sin } (\omega t) \tag{5.4}$$

where $\delta\alpha$ and ω are the amplitude and frequency of the harmonic oscillation respectively. Figure 17 shows the results of the 8.4% thick Von Mises symmetric airfoil oscillating at an amplitude of 0.1 radian at a reduced frequency of $\omega c/V_\infty = 20.0$ about the leading edge. Again the plots are given together with the single airfoil case undergoing the same motion for comparison purposes.



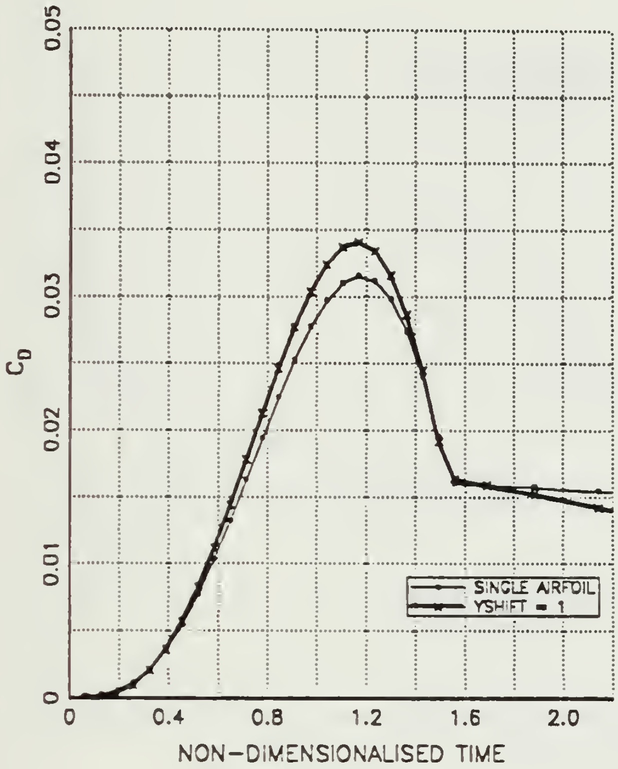
(a) Time History of the Lift Coefficients.

Figure 15. **Modified Ramp Change in Angle Of Attack:** Ramp change in AOA for an 8.4 per-cent thick Von Mises airfoils placed at 1 chord length vertical distance with initial AOA = 0.0 radians and final AOA = 0.1 radians, rise time of 1.5 pivoting at the mid chord.



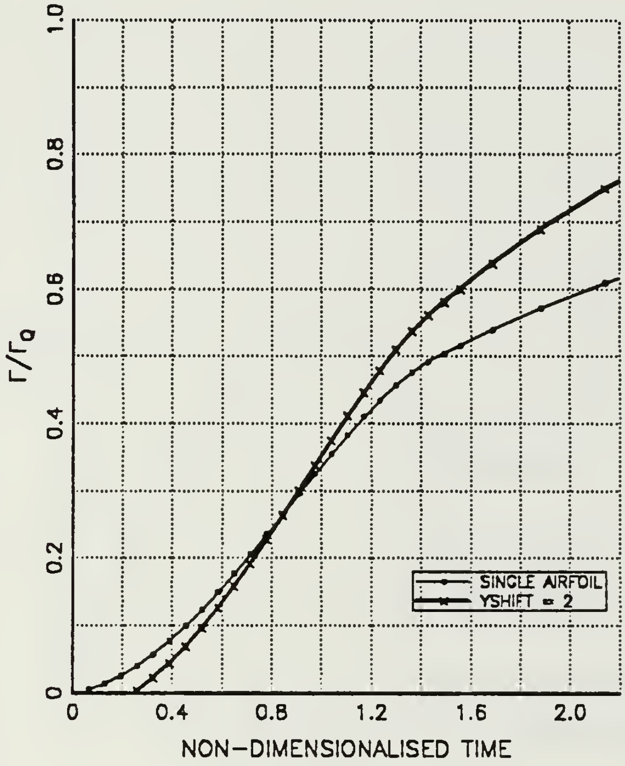
(b) Time History of the Moment Coefficients.

Figure 15 (Cont'd)



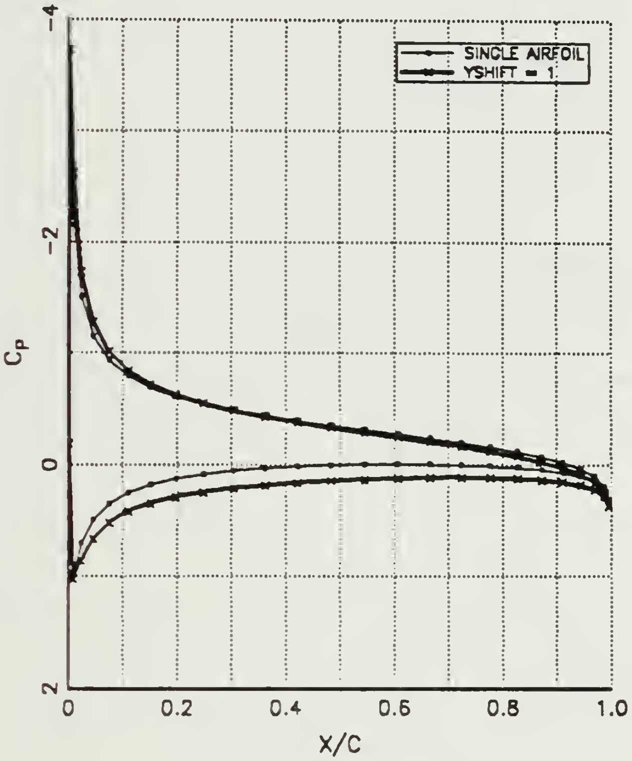
(c) Time History of the Drag Coefficients.

Figure 15 (Cont'd)



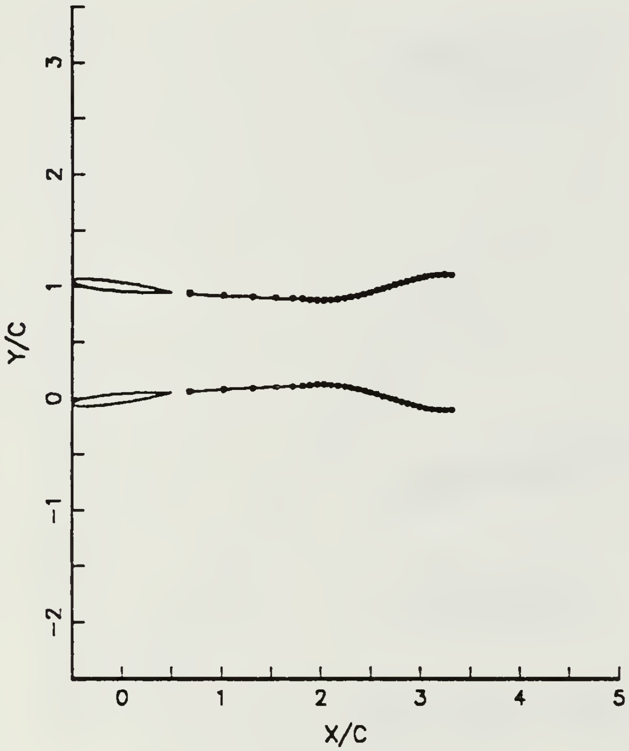
(d) Time History of the Circulation

Figure 15 (Cont'd)



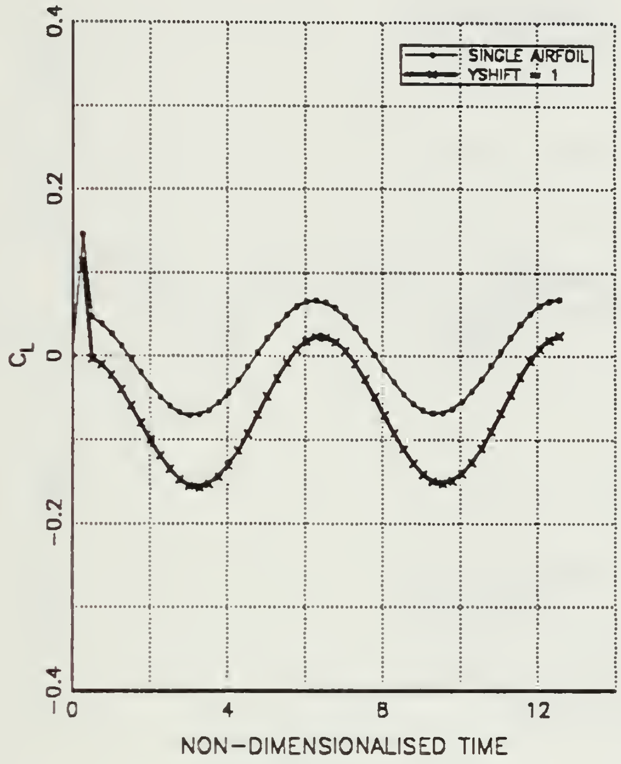
(e) Pressure Distribution at $tV_\infty/c = 2.3$

Figure 15 (Cont'd)



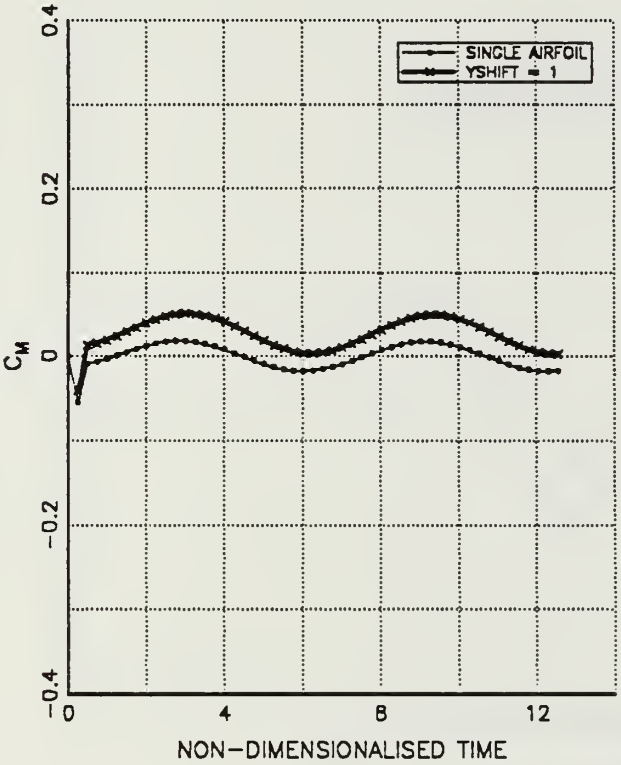
(f) Wake pattern at $tV_\infty/c = 2.3$

Figure 15 (Cont'd)



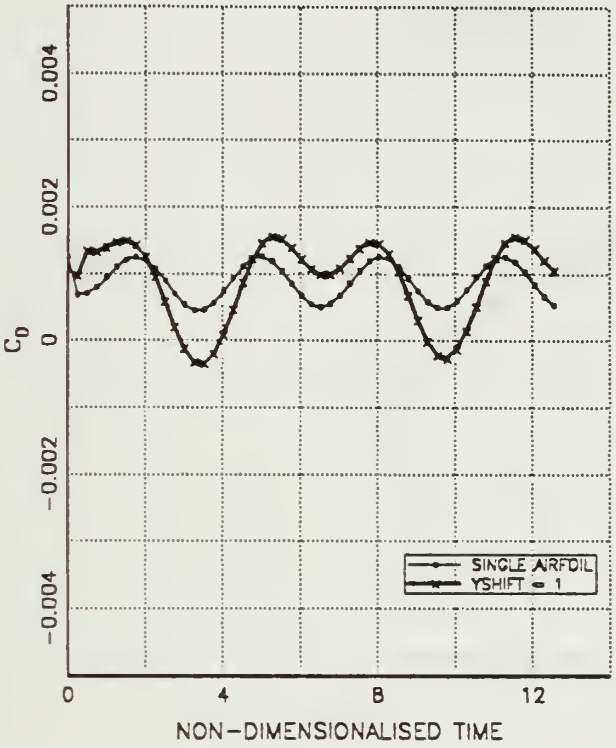
(a) Time History of the Lift Coefficients.

Figure 16. **Harmonic Plunging Motion:** Translational harmonic plunging AOA for NACA-0015 airfoils placed at 1 chord length vertical distance set at AOA = 0.0 radians with plunging amplitude of 0.018 chord length at a reduced frequency of 1.



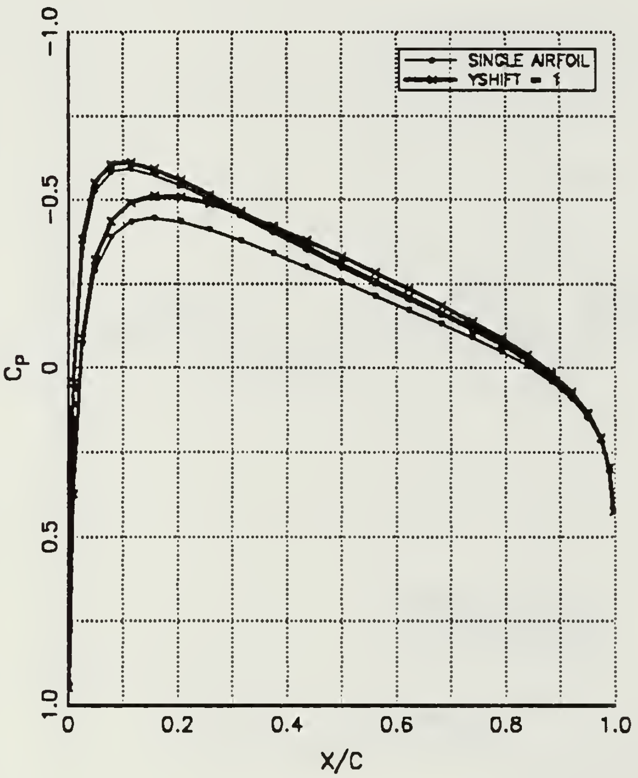
(b) Time History of the Moment Coefficients.

Figure 16 (Cont'd)



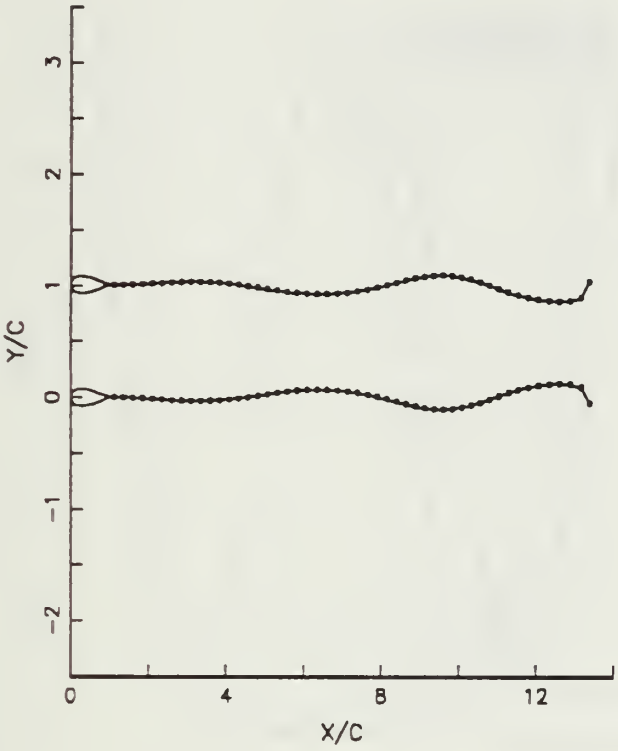
(c) Time History of the Drag Coefficients.

Figure 16 (Cont'd)



(d) Pressure Distribution at $tV_\infty/c = 12.6$

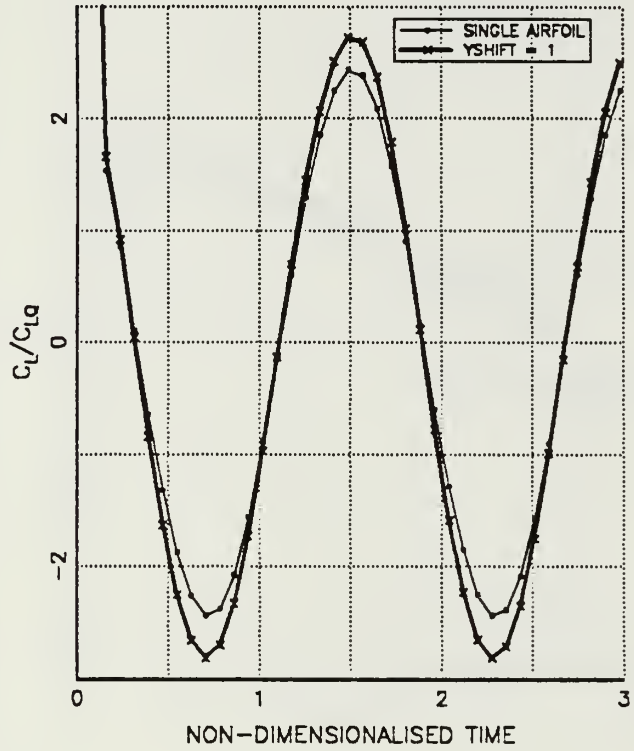
Figure 16 (Cont'd)



Note : Different scales for x and y axes

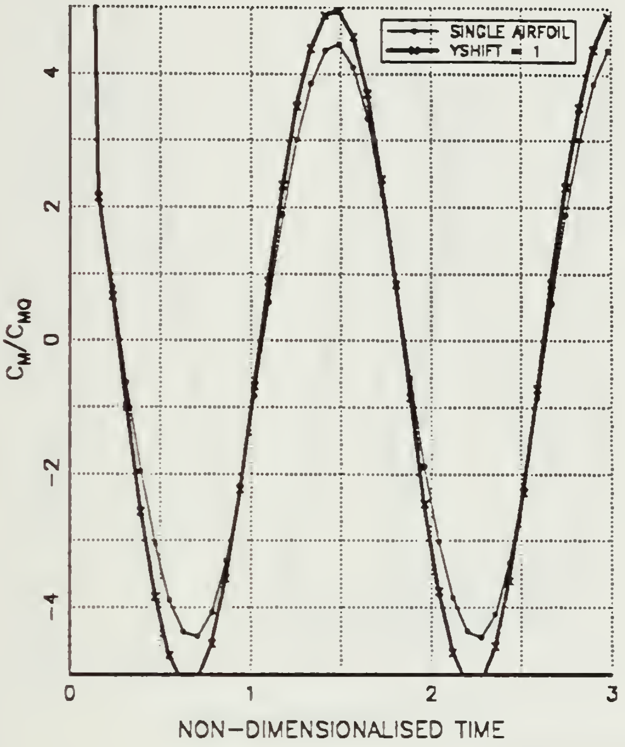
(e) Wake Pattern at $tV_\infty/c = 12.6$

Figure 16 (Cont'd)



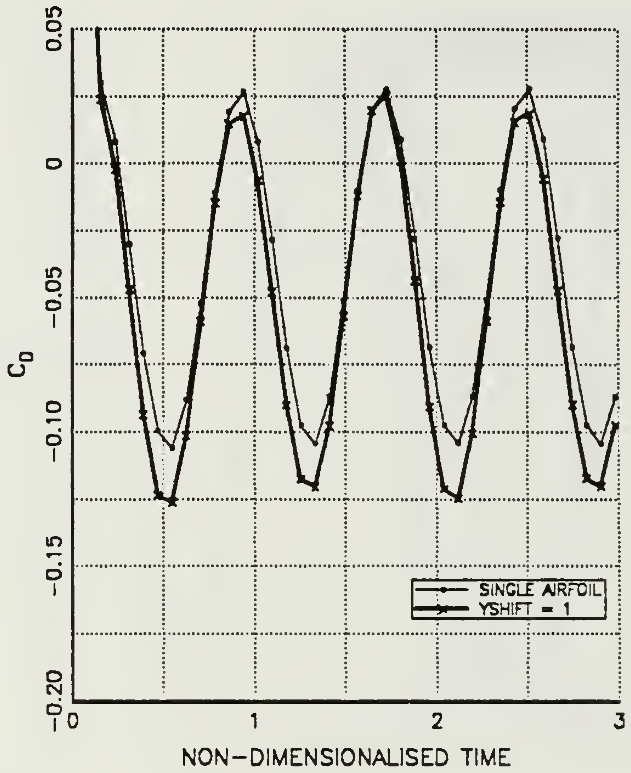
(a) Time History of the Lift Coefficients.

Figure 17. Harmonic Pitching Motion: Rotational harmonic pitching AOA for NACA-0015 airfoils placed at 1 chord length vertical distance set at AOA = 0.0 radians with pitching amplitude of 0.1 radian at a reduced frequency of 4 pivoting about the leading edges.



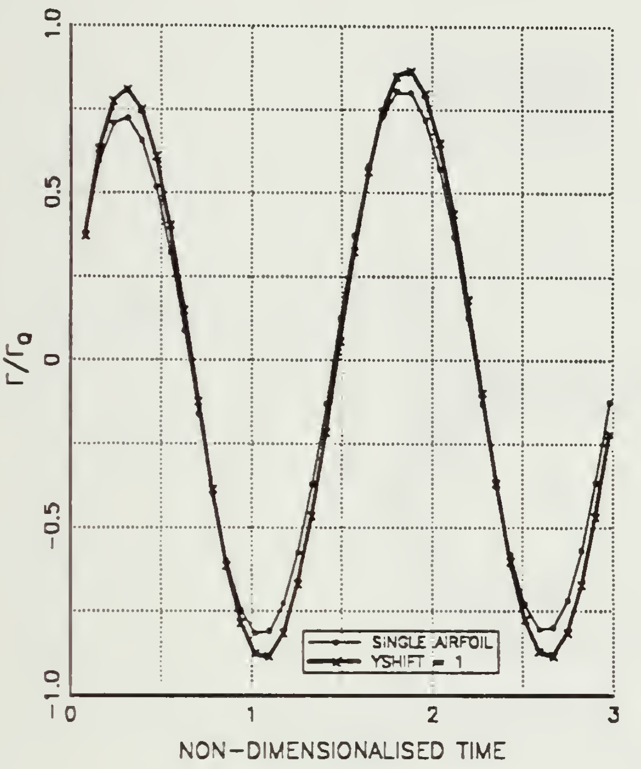
(b) Time History of the Moment Coefficients.

Figure 17 (Cont'd)



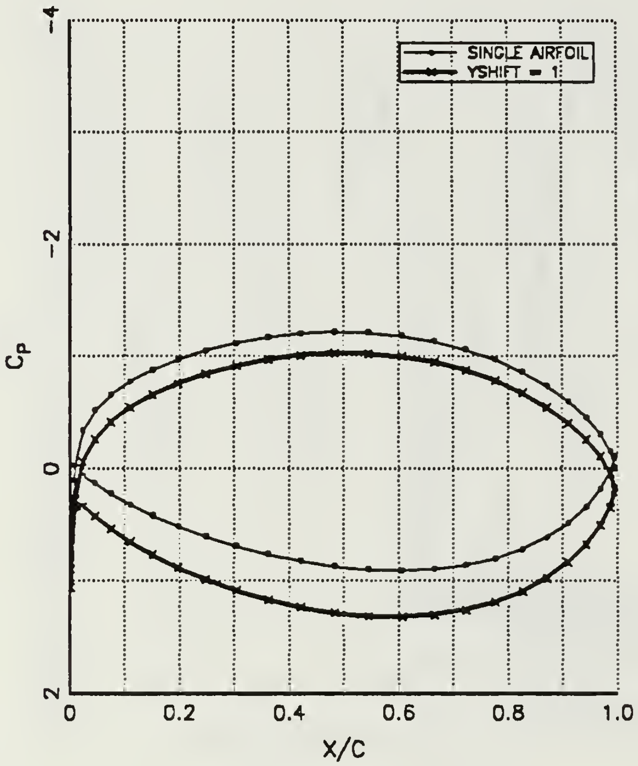
(c) time History of the Drag Coefficients.

Figure 17 (Cont'd)



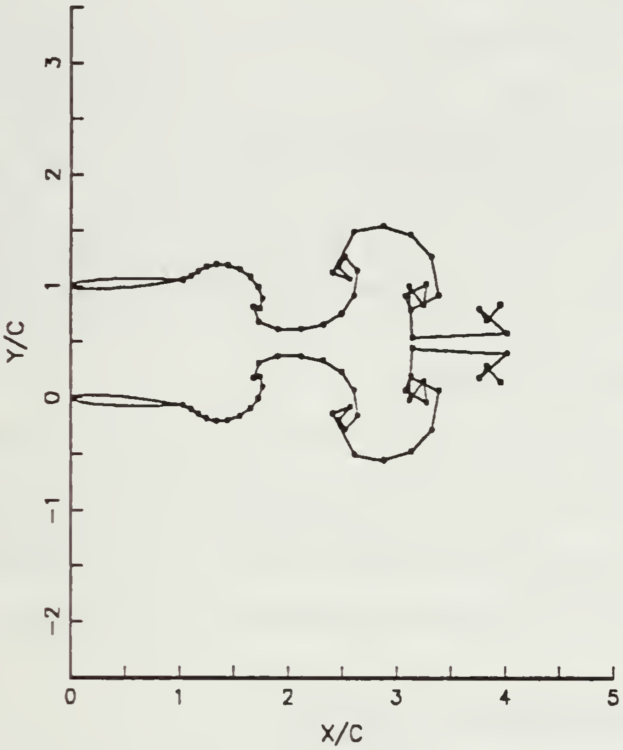
(d) Time History of the Circulation.

Figure 17 (Cont'd)



(e) Pressure Coefficient at $tv_\infty = 3.0$.

Figure 17 (Cont'd)



(f) Wake pattern at $tv_\infty = 3.0$.

Figure 17 (Cont'd)

VI. CONCLUSION

A. GENERAL COMMENTS

USPOTF2 has been developed as an intermediate stage to getting a solution for a full cascade undergoing unsteady motion. In itself, it can be used to simulate unsteady wing--tail, wing--canard, wing--flap, wing--aileron, horizontal tail--elevator, vertical tail--rudder and a whole host of moving--stationary airfoil interactions. These simulations require a little addition to the main program and would appear as modules modelling the variation of the global angles of attack and relative displacements as per all the sub-cases done. The subroutines would not be affected by the severity of the test case except for possibly subroutine NEWPOS.

Validation of USPOTF2 has been done against Giesing's code for the particular case of a step change in angle of attack. This was shown to have good correlation. However, it can not be said that the agreement will hold for other angles of attack and more sub-case runs for smaller angles of attack should be made. In the same manner, the success of the step change in AOA in no way validates the other sub-cases which, for the time being, remain unproven.

B. ENHANCING USPOTF2 PROGRAM CAPABILITY

As noted above, USPOTF2 needs to be tried more extensively either with the existing sub-cases or with new sub-cases against existing numerical or experimental results.

The software in itself has an implicit weakness in the numerical computation of the velocity potential. The velocity potential at the leading edge is obtained by integrating the velocity field from the leading edge to a point 100 chord lengths upstream of the leading edge. Two assumptions are made: First, the disturbance velocity at the point 100 chord lengths of the leading edge approaches zero. Second, the contribution to the velocity potential due to the integration from the point 100 chord lengths upstream of the leading edge to upstream infinity must not change in time. The coefficients of pressure will be accurate only, if those two assumptions hold (at least in an approximate sense). This however does not give a very 'exact' solution as by increasing the chord length by 900 per-cent to 1000 chord lengths, there is a change in the pressure coefficient at the trailing edge of about 8 per-cent for the first time step. Krainer has implemented an analytical solution to the potential problem for the single airfoil. A similar procedure to upgrade USPOTF2 should be considered.

Other improvements to the code would be the reduction of redundant computations. An obvious example would be the subroutines VELDIS and PRESS which essentially do the same work for the steady and unsteady flow respectively. Again Krainer [Ref. 6] has improved the original code for the single airfoil (U2DIIF) and though some of his improvements were implemented in USPOTF2, there still remains a task to do the full job completely.

Finally, the original primary objective of extending the code to solve for the unsteady flow solution for 3,4 airfoils and leading to a full cascade still remains a big task, not just for the programmer but also for the computer system in terms of computational storage and time requirements.

APPENDIX A. USPOTF2 SOURCE LISTINGS

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C                      PROGRAM  USPOTF2                                C
C                      V E R S I O N  1                                C
C                        SEPTEMBER 88                                  C
C                                                                      C
C        UNSTEADY MOTION FOR TWO AIRFOILS IN POTENTIAL                 C
C                   INCOMPRESSIBLE  FLOW                               C
C        USING PANEL METHODS BASED ON THE HESS & SMITH                 C
C           AND WAKE MODEL BASED ON BASU & HANCOCK                     C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +             COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +             NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +             COSTHL(201),SINTHL(201)
      COMMON /WAK/ VYW(2),VXW(2),WAKE(2),DT
      COMMON /WAK2/ VYWK(2),VXWK(2)
      COMMON /SING/ Q(200),GAMMA(2),QK(200),GAMK(2)
      COMMON /CORV/ CV(2,200),XC(2,200),YC(2,200),M,TD,CVVX(2,200),
     +              CVVY(2,200),XCI(2,200),YCI(2,200)
      COMMON /POT/ PHI(200),PHIK(200)
      COMMON /GUST/ UG(200),VG(200),XGF,UGUST,VGUST
      COMMON /EXTV/ UE(200),VN(200)
      COMMON /PARD/ NACAD(2),TAUD(2),EPSMAD(2),PTMAXD(2)
      COMMON /COF/ A(201,211),KEQNS
      COMMON /GEOM/ SINALF(2),COSALF(2),OMEGA(2),UX(2),UY(2),PIVOT,
     +              XPRM,YPRM
      COMMON /CPD/ CP(200),SCL,T,SCM,SGAM
      COMMON /GIES/ NGIES
      DIMENSION XXC(2,200),YYC(2,200),TOL1(2),TOL2(2),THENP1(2),
     +SINANG(2),COSANG(2),COSDA(2),SINDA(2),DHX(2),DHY(2),ALP(2),
     +ALPHA(2),ALPI(2),ANGLE(2),DELHX(2),DELHY(2),PHASE(2),PHA(2),
     +HX(2),HXO(2),HY(2),HYO(2)
C
C     INITIALISATION
C
      PI      = 3.1415926585
      M       = 0
      XPRM    = 0.0
      YPRM    = 0.0
C
C     INPUT FROM FILE CODE 5 AND SET UP LOCAL PANEL NODES AND SLOPES
C
      WRITE (6,1003)
 1003 FORMAT (/////,' DATA READ FROM FILE CODE 1',//)
      CALL INDATA
      CALL SETUP
      READ (1,502) ALPI(1),ALPI(2),DALP,TCON,FREQ,PIVOT,IPHASE
      WRITE (6,502) ALPI(1),ALPI(2),DALP,TCON,FREQ,PIVOT,IPHASE
 501  FORMAT (8F10.6)
 502  FORMAT (6F10.6,I3)
      READ (1,501) UGUST,VGUST,DELHX(1),DELHX(2),DELHY(1),DELHY(2),
     +             PHASE(1),PHASE(2)
      WRITE (6,501) UGUST,VGUST,DELHX(1),DELHX(2),DELHY(1),DELHY(2),
     +             PHASE(1),PHASE(2)

      READ (1,503) TF,DTS,TOL,TADJ,SCL,SCM,SGAM,NGIES
      WRITE (6,503) TF,DTS,TOL,TADJ,SCL,SCM,SGAM,NGIES
 503  FORMAT (7F10.6,I3)
      IF (IFLAG .EQ. 0) WRITE (6,1005)
 1005 FORMAT (///,' COORDINATES OF AIRFOIL NODES',
     + //,3X,' X/C',6X,' Y/C',/)
      IF (IFLAG .EQ. 0) WRITE (6,1010) (XI(I),YI(I),I=1,2*(NODTOT+1))
 1010 FORMAT (F10.6,F10.6)
      WRITE (6,1000) NAIRFO
 1000 FORMAT(//,'TOTAL NO OF AIRFOILS = ',I4,/)
C
C     STEADY FLOW CALCULATION AT ALPI(I)
C
C     INITIAL DEFINITION
C
      NP1     = NODTOT + 1
      NP2     = 2 * NP1 + 1
      NP3     = NAIRFO*NODTOT+1
      NP4     = NP3+1
      NP5     = NP4+1
      DO 101 I = 1,2
      ALPHA(I) = ALPI(I)
      IF (ALPHA(I) .GT. 90.)    GO TO 200
      COSALF(I) = COS(ALPHA(I)*PI/180.)
 101  SINALF(I) = SIN(ALPHA(I)*PI/180.)
      CALL NEWPOS(0)
      DO 1100 L = 1,NAIRFO
 1100 WRITE (6,1020) L,SS(L)
      WRITE (6,1040) XSHIFT,YSHIFT
 1020 FORMAT(//,' AIRFOIL(',I2,') PERIMETER LENGTH = ',F10.6,/)
 1040 FORMAT(//,' XSHIFT = ',F5.1,'   YSHIFT = ',F5.1,/)
      DO 102 I = 1,2
 102  WRITE (6,1030) I,ALPHA(I)
 1030 FORMAT (//,' STEADY FLOW SOLUTION AT ALPHA(',I2,') = ',F10.6,/)
      IF (SCL.NE.0)  WRITE (8,1035)
 1035 FORMAT(3X,'TIME',4X,'CL(1)/SCL',1X,'CL(2)/SCL',1X,'CM(1)/SCM',
     +1X,'CM(2)/SCM',3X,'CD(1)',4X,'CD(2)',4X,
     +'GAMK(1)/SG',1X,'GAMK(2)/SG',//)
      IF (SCL.EQ.0)  WRITE (8,1036)
 1036 FORMAT(3X,'TIME',6X,'CL(1)',5X,'CL(2)',5X,'CM(1)',
     +5X,'CM(2)',5X,'CD(1)',4X,'CD(2)'//)
      CALL INFL (0)
      CALL COEF (0)
      CALL GAUSS(3,0,0)
      CALL KUTTA(NITR,PVTAG)
      CALL VELDIS
      SIN1 = SINALF(1)
      SIN2 = SINALF(2)
      COS1 = COSALF(1)
      COS2 = COSALF(2)
      CALL FANDM(SIN1,SIN2,COS1,COS2)
C
C     INITIALISATION FOR UNSTEADY FLOW CALCULATION TO BEGIN
C
      DA      = 0.0
      XGF     = 0.0
      DO 100  L  = 1,NAIRFO
      HX(L)   = 0.0
      HY(L)   = 0.0
      HXO(L)  = 0.0
      HYO(L)  = 0.0
      ANGLE(L)     = ALPI(L)*PI/180.  + ATAN(VGUST/(1.+UGUST))
      OMEGA(L)     = 0.0
      ALP(L)       = ALPI(L)
      DHX(L)       = 0.0
      DHY(L)       = 0.0
      COSANG(L)    = COS(ANGLE(L))
      SINANG(L)    = SIN(ANGLE(L))
      UX(L)        = 0.0
      UY(L)        = 0.0
      COSDA(L)     = 1.0
      SINDA(L)     = 0.0
      KIG          = (L -1)*NODTOT
      VXW(L) = COSALF(L)
      VYW(L) = SINALF(L)
      GAMK(L)     = GAMMA(L)
      PHA(L)  = PHASE(L)*PI/180.
      DO 100  IG = 1,NODTOT
      UG(IG+KIG)  = 0.0
  100 VG(IG+KIG)  = 0.0
      T       = 0.0
      TOLD    = 0.0
C
C        RIGID BODY MOTIONS OF AIRFOIL
C
      IF (FREQ .NE.  0.0) GO TO 1
      IF (DALP .EQ.  0.0) GO TO 2
      IF (TCON .NE.  0.0) GO TO 3
      IF (IPHASE .EQ.  0) GO TO 4
      ALPHA(1)= ALPI(1) + DALP
      ALPHA(2)= ALPI(2) + DALP
      GO TO 5
 4    ALPHA(1)= ALPI(1) + DALP
      ALPHA(2)= ALPI(2) - DALP
 5    DO 6 I = 1,2
      COSALF(I)  = COS(ALPHA(I)*PI/180.)
      SINALF(I)  = SIN(ALPHA(I)*PI/180.)
 6    CONTINUE
      CALL NEWPOS(0)
 3    DT      = DTS
      TD      = DTS
      GO TO 60
 2    IF ((UGUST .EQ.  0.0) .AND.  (VGUST .EQ.  0.0)) GO TO 200
      DT      = DTS
      TD      = DTS
      GO TO 60
 1    DT       = 2.0*PI/(FREQ*DTS)
      TD       = DT
 60   T        = DT
      WRITE (6,1051)
 1051 FORMAT (/////,' ***************************************',/,
     + ' ***  BEGIN UNSTEADY FLOW SOLUTION  ****',/,
     + ' ***************************************')
      TRY      = 0
 40   M        = M + 1
      IF (T .GT. TF) GO TO 200
C
C        STORE CORE VORTEX COORDINATES FOR TIME STEP ADJUSTMENTS
C
      IF (M .EQ. 1) GO TO 50
      DO 51   L = 1,NAIRFO
      DO 51   I = 1,M-1
      XXC(L,I)  = XCI(L,I)
 51   YYC(L,I)  = YCI(L,I)
 50   IF (FREQ .NE. 0.0) GO TO 11
      IF (DALP .EQ. 0.0) GO TO 22
      IF (TCON .NE. 0.0) GO TO 33
C
C        STEP CHANGE IN AOA
C
      IF (TADJ .NE. 0.0) GO TO 70
C     IF INCREMENTAL PROGRESSIVE TIME STEP IS REQUIRED
C     USE ... TD = FLOAT(M+1)*DTS ... OTHERWISE CONSTANT TIME STEP
      TD       = DTS
      GO TO 70
C
C        MODIFIED RAMP CHANGE IN AOA
C
 33   IF (T .GT. TCON) GO TO 34
      DAL      = DALP * (3.-2.*T/TCON)*(T/TCON)**2
      OMEGA(1)   = - (DALP*PI/180.) * (6.*T/(TCON*TCON)) * (1.-T/TCON)
      IF (IPHASE .EQ. 0) GO TO 41
      ALPHA(1)= ALPI(1) + DAL
      ALPHA(2)= ALPI(2) + DAL
      OMEGA(2)   = OMEGA(1)
      GO TO 42
 41   ALPHA(1)= ALPI(1) + DAL
      ALPHA(2)= ALPI(2) - DAL
      OMEGA(2)   = - OMEGA(1)
 42   DO 43  I = 1,2
      COSALF(I)   = COS(ALPHA(I)*PI/180.)
      SINALF(I)   = SIN(ALPHA(I)*PI/180.)
      DA         = ALPHA(I) - ALP(I)
      COSDA(I)   = COS(DA*PI/180.)
      SINDA(I)   = SIN(DA*PI/180.)
      DHX(I)     = PIVOT * (1.-COSDA(I))
      DHY(I)     = - PIVOT * SINDA(I)
      UY(I)      = PIVOT * OMEGA(I)
 43   CONTINUE
      MTCON   = M
      CALL NEWPOS(0)
      GO TO 70
 34   DAL     = 0.0
      IF (IPHASE .EQ. 0) GO TO 45
      ALPHA(1)= ALPI(1) + DALP
      ALPHA(2)= ALPI(2) + DALP
      GO TO 46
 45   ALPHA(1)  = ALPI(1) + DALP
      ALPHA(2)  = ALPI(2) - DALP
 46   DO 44 I = 1,2
      COSALF(I) = COS(ALPHA(I)*PI/180.)
      SINALF(I) = SIN(ALPHA(I)*PI/180.)
      DA        = 0.0
      COSDA(I)  = 1.0
      SINDA(I)  = 0.0
      OMEGA(I)  = 0.0
      DHX(I)    = 0.0
      DHY(I)    = 0.0
      UX(I)     = 0.0
      UY(I)     = 0.0
 44   CONTINUE
      CALL NEWPOS(0)
      IF (TADJ .NE. 0.0) GO TO 70
      TD      = FLOAT(M+1-MTCON)*DTS
      GO TO 70
C
C        SHARP EDGE GUST (UGUST AND/OR VGUST) NOT PROVEN DUE TO
C          INCONSISTENT ASSUMPTIONS REQUIRING ROTATIONAL FLOW
C
C
 22   XGF     = T
      DO 113  L = 1,NAIRFO
      LIG     = (L-1)*NODTOT
      KIG     = (L-1)*NP1
      DO 110  IG = 1,NODTOT
      UG(IG+LIG)  = 0.0
      VG(IG+LIG)  = 0.0
      XG      = X(IG+KIG)
      XGP1    = X(IG+KIG+1)
      IF (IG .LT. NLOWER+1) GO TO 120
      IF (XGF .LE. XG) GO TO 110
      IF (XGF .GE. XGP1) GO TO 111
      FAC     = (XGF - XG)/(XGP1 - XG)
      UG(IG+LIG)  = UGUST*FAC
      VG(IG+LIG)  = VGUST*FAC
      GO TO 110
 111  UG(IG+LIG)  = UGUST
      VG(IG+LIG)  = VGUST
      GO TO 110
 120  IF (XGF .LE. XGP1) GO TO 110
      IF (XGF .GE. XG) GO TO 121
      FAC     = (XGF - XGP1)/(XG - XGP1)
      UG(IG+LIG)  = UGUST*FAC
      VG(IG+LIG)  = VGUST*FAC
      GO TO 110
 121  UG(IG+LIG)  = UGUST
      VG(IG+LIG)  = VGUST
 110  CONTINUE
 113   CONTINUE
       IF (XGF .LE. COSALF(L)) MGUST = M
       IF (TADJ .NE. 0.0) GO TO 70
       IF (XGF .GT. COSALF(L)) TD = FLOAT(M+1-MGUST)*DTS
       GO TO 70
C
C         TRANSLATION HARMONIC OSCILLATION
C
 11    IF (DALP .NE. 0.0) GO TO 12
       DO 131 I = 1,NAIRFO
       HX(I)   = DELHX(I) * SIN(FREQ*T + PHA(I))
       HY(I)   = DELHY(I) * SIN(FREQ*T)
       DHX(I)  = HX(I) - HXO(I)
       DHY(I)  = HY(I) - HYO(I)
       UX(I)   = DELHX(I)*FREQ*COS(FREQ*T+PHA(I))
       UY(I)   = DELHY(I)*FREQ*COS(FREQ*T)
 131   CONTINUE
       XPRM    = DHX(2) - DHX(1)
       YPRM    = DHY(2) - DHY(1)
       CALL NEWPOS(0)
       GO TO 70
C
C         ROTATIONAL HARMONIC OSCILLATION
C
 12    DAL     = DALP*SIN(FREQ*T)
       OMEGA(1)   = - (DALP*PI/180.) * FREQ * COS(FREQ*T)
       IF (IPHASE .EQ. 0) GO TO 141
       ALPHA(1)= ALPI(1) + DAL
       ALPHA(2)= ALPI(2) + DAL
       OMEGA(2)   = OMEGA(1)
       GO TO 142
 141   ALPHA(1)= ALPI(1) + DAL
       ALPHA(2)= ALPI(2) - DAL
       OMEGA(2)   = - OMEGA(1)
 142   DO 143  I = 1,2
       COSALF(I)   = COS(ALPHA(I)*PI/180.)
       SINALF(I)   = SIN(ALPHA(I)*PI/180.)
       DA          = ALPHA(I) - ALP(I)
       COSDA(I)    = COS(DA*PI/180.)
       SINDA(I)    = SIN(DA*PI/180.)
       DHX(I)      = PIVOT * (1.-COSDA(I))
       DHY(I)      = - PIVOT * SINDA(I)
       UY(I)       = PIVOT * OMEGA(I)
 143   CONTINUE
       CALL NEWPOS(0)
C
C         TRANSFORM CORE VORTEX COORDINATES W. R. T. NEW AIRFOIL POSITION
C
 70    IF (M .EQ. 1) GO TO 80
       DO 85   L = 1,NAIRFO
       DO 90   I = 1,M-1
       XCI(L,I)    = XXC(L,I) + CVVX(L,I) * DT
       YCI(L,I)    = YYC(L,I) + CVVY(L,I) * DT
       XCO      = XCI(L,I)
       YCO      = YCI(L,I)
       XCI(L,I)    = XCO*COSDA(L) - YCO*SINDA(L) + DHX(L)
 90   YCI(L,I)    = XCO*SINDA(L) + YCO*COSDA(L) + DHY(L)
 85   CONTINUE
      CALL NEWPOS(3)
 80   CONTINUE
      WRITE (6,1001) T,DT
 1001 FORMAT (/////,' TIME STEP TK = ',F10.6,10X,'TK - TKM1 = ',F10.6,/)
      WRITE (6,1004) ALPHA(1),ALPHA(2),OMEGA(1),OMEGA(2),UX(1),UX(2),
     +               UY(1),UY(2)
 1004 FORMAT (/,' ALPHA(1)  = ',F10.6,5X,' ALPHA(2)  = ',F10.6,/,
     +' OMEGA(1)  = ',F10.6,5X,' OMEGA(2)  = ',F10.6,
     +/,' U(1)      = ',F10.6,5X,'  U(2)     = ',
     +F10.6,/,' V(1)      = ',F10.6,5X,'  V(2)     = ', F10.6,///,
     + 1X,' NITR   VXW(1)    VYW(1)   WAKE(1)  THETA(1)     GAMK(1)
     + VXW(2)    VYW(2)   WAKE(2)   THETA(2)   GAMK(2)   ',/)
C
C     CALCULATE THE TRAILING EDGE WAKE ELEMENT
C
      NUM     = 0
      NITR    = 0
 10   DO 15 L      = 1,NAIRFO
      KI           = (L-1)*(NODTOT+1)
      WAKE(L) = SQRT(VYW(L)*VYW(L)+VXW(L)*VXW(L))*DT
      THENP1(L) = ATAN2(VYW(L),VXW(L))
      COSTHL(NP1+KI) = COS(THENP1(L))
 15   SINTHL(NP1+KI) = SIN(THENP1(L))
      WRITE (6,1002) NITR,VXW(1),VYW(1),WAKE(1),THENP1(1),GAMK(1),
     +VXW(2),VYW(2),WAKE(2),THENP1(2),GAMK(2)
 1002 FORMAT (I5,4F10.6,E14.6,4F10.6,E14.6)
      XI(NP2)  = XI(NP1) + WAKE(1)*COSTHL(NP1)
      YI(NP2)  = YI(NP1) + WAKE(1)*SINTHL(NP1)
      XI(NP2+1)  = XI(NP1) + WAKE(2)*COSTHL(2*NP1)
      YI(NP2+1)  = YI(NP1) + WAKE(2)*SINTHL(2*NP1)
      CALL NEWPOS(1)
      CALL INFL (NITR)
      CALL COEF (NITR)
      CALL GAUSS(3,M,NITR)
C     WRITE (6,*) 'A(I,NP)',(A(I,101),I=1,100)
C     WRITE (6,*) 'A(I,NTOT)',(A(I,102),I=1,100)
C     WRITE (6,*)'BEFORE KUTTA'
      CALL KUTTA(NITR,PVTAG)
      IF (PVTAG .LT. 0.01) GOTO 13
      NUM = NUM + 1
C     WRITE (6,*)'NUM =',NUM
      IF (NUM .GT. 1) GOTO 13
      DO 7 I = 1,2
      VXW(I) = 1
  7   VYW(I) = 0
      GOTO 10
 13   CALL TEWAK
      DO 24 L = 1,NAIRFO
      TOL1(L) = ABS(VYW(L) - VYWK(L))/VYWK(L)
      TOL2(L) = ABS(VXW(L) - VXWK(L))/VXWK(L)
      VYW(L)  = VYWK(L)
 24   VXW(L)  = VXWK(L)
      IF ((TOL1(1) .LT. TOL) .AND. (TOL2(1) .LT. TOL)
     +.AND. (TOL1(2) .LT. TOL) .AND. (TOL2(2) .LT. TOL)) GO TO 20
```

```
      IF (NITR .GT. 25) THEN
      TOL     = TOL*10
      WRITE (6,1023) TOL
      ENDIF
      IF (NITR .GT. 50) STOP
      NITR    = NITR + 1
      GO TO 10
 20   WRITE (6,1011) NITR
 1011 FORMAT (//,' CONVERGED SOLUTION OBTAINED AFTER NITR = ',I3)
 1023 FORMAT (//,' ***** TOLERANCE CRITERIA CHANGED : TOL = ',F10.6)
      CALL PRESS
      IF ((UGUST .EQ. 0.0) .AND. (VGUST .EQ. 0.0)) GO TO 300
      SIN1 = SINANG(1)
      SIN2 = SINANG(2)
      COS1 = COSANG(1)
      COS2 = COSANG(2)
      CALL FANDM(SIN1,SIN2,COS1,COS2)
      GO TO 400
 300  SIN1 = SINALF(1)
      SIN2 = SINALF(2)
      COS1 = COSALF(1)
      COS2 = COSALF(2)
      CALL FANDM(SIN1,SIN2,COS1,COS2)
 400  CONTINUE
C
C     ADJUST TIME STEP (TADJ .NE. 0.0) IF NECESSARY
C
      IF (TADJ .EQ. 0.0) GO TO 95
      WRITE (5,2001)
 2001 FORMAT (//,' DO YOU WANT TO ADJUST TIME STEP ? 0 - NO, 1 - YES')
      READ (5,*) IDT
      IF (IDT .EQ. 0) GO TO 95
      DT       = TADJ * DT
      T        = TOLD + DT
      WRITE (6,1006)
 1006 FORMAT (//,' BACK-TRACK COMPUTATION AND ADJUST TIME-STEP',//)
C
C        WAKE ELEMENT LEAVES TRAILING EDGE AS A CORE-VORTEX
C
 95   DO 96  L = 1,NAIRFO
      CV(L,M)     = SS(L)*(GAMMA(L)-GAMK(L))
      CVVX(L,M)   = VXW(L)
 96   CVVY(L,M)   = VYW(L)
      XCI(1,M)     = XI(NP1) + 0.5*WAKE(1)*COSTHL(NP1)
      YCI(1,M)     = YI(NP1) + 0.5*WAKE(1)*SINTHL(NP1)
      XCI(2,M)     = XI(NP1) + 0.5*WAKE(2)*COSTHL(NP1*2)
      YCI(2,M)     = YI(NP1) + 0.5*WAKE(2)*SINTHL(NP1*2)
      CALL NEWPOS(2)
      WRITE (6,1052)
 1052 FORMAT (//,' TRAILING VORTICES DATA',//,
     + 4X,'M',4X,'X1I(M)',5X,'Y1I(M)',6X,'X1(M)',6X,'Y1(M)',
     + 5X,'CIRC1',6X,'X2I(M)',5X,'Y2I(M)',
     + 5X,'X2(M)',6X,'Y2(M)',6X,'CIRC2',/)
      DO 900  I = 1,M
 900  WRITE (6,1050) I,XCI(1,I),YCI(1,I),XC(1,I),YC(1,I),CV(1,I),
     + XCI(2,I),YCI(2,I),XC(2,I),YC(2,I),CV(2,I)
 1050 FORMAT(I5,10F11.6)
      CALL CORVOR
C
C        RE-INITIALISE PARAMETERS FOR NEXT TIME STEP CALCULATION
C
      DO 30  L = 1,NAIRFO
      LI       = (L-1)*NODTOT
      HXO(L)  = HX(L)
      HYO(L)  = HY(L)
      GAMMA(L)= GAMK(L)
      ALP(L)  = ALPHA(L)
      DO 30   I = 1,NODTOT
      PHI(I+LI)  = PHIK(I+LI)
 30   CONTINUE
      TOLD     = T
      DT       = TD
      T        = T + TD
      GO TO 40
 200  WRITE (6,1024) TOL
 1024 FORMAT (1X,/'***** TOLERANCE CRITERION USED: TOL ',F10.6)
      STOP
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     SUBROUTINE INDATA                                                C
C                                                                      C
C                    SET PARAMETERS OF BODY SHAPE                      C
C                    FLOW SITUATION, AND NODE DISTRIBUTION             C
C                                                                      C
C                    USER MUST INPUT                                   C
C                         NLOWER = NUMBER OF NODES ON LOWER SURFACE    C
C                         NUPPER = NUMBER OF NODES ON UPPER SURFACE    C
C                    PLUS DATA ON BODY AND SUBROUTINE BODY             C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     NACAD(I) .... NACA NUMBERS FOR THE TWO AIRFOILS
C     TAUD(I)  .... MAX THICKNESS FOR THE TWO AIRFOILS
C     EPSMAD(I) ... MAX CAMBER FOR THE TWO AIRFOILS
C     PTMAXD(I) ... CHORDWISE POSN FOR MAX CAMBER FOR THE TWO AIRFOILS
C     _________________________________________________________________
      SUBROUTINE INDATA
      DIMENSION TITLE(20)
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +             COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +             NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +             COSTHL(201),SINTHL(201)
      COMMON /PARD/ NACAD(2),TAUD(2),EPSMAD(2),PTMAXD(2)
      READ (1,501) ITITLE
      WRITE (6,501) ITITLE
      DO 10   I = 1,ITITLE
      READ (1,502) TITLE
 10   WRITE (6,503) TITLE
 501  FORMAT(3I5)
 502  FORMAT(20A4)
 503  FORMAT(1X,20A4)
 504  FORMAT(I5,2F10.6)
      READ (1,501) IFLAG,NLOWER,NUPPER
      WRITE (6,501) IFLAG,NLOWER,NUPPER
      READ (1,504) NAIRFO,XSHIFT,YSHIFT
      WRITE (6,504) NAIRFO,XHIFT,YSHIFT
      IF (IFLAG .NE. 0) RETURN
C
C     COMPUTATION FOR NACA 4-DIGITS SERIES
C
      READ (1,501) NACAD(1)
      WRITE (6,501) NACAD(1)
      READ (1,501) NACAD(2)
      WRITE (6,501) NACAD(2)
      DO 100 I=1,NAIRFO
      IEPS      = NACAD(I)/1000
      IPTMAX    = NACAD(I)/100 - 10*IEPS
      ITAU      = NACAD(I) - 1000*IEPS - 100*IPTMAX
      EPSMAD(I) = IEPS*0.01
      PTMAXD(I)  = IPTMAX*0.1
      TAUD(I)    = ITAU*0.01
      IF (IEPS .LT. 10) GOTO 100
C
C     COMPUTATION FOR NACA 5-DIGITS SERIES NOTING THAT TAUD(I) IS
C     COMPUTED AS PER 4-DIGITS SERIES.
C
      PTMAXD(I)  = 0.2025
      EPSMAD(I) = 2.6595*PTMAXD(I)**3
 100  CONTINUE
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     SUBROUTINE SETUP                                                 C
C                                                                      C
C             SETUP COORDINATES OF PANEL NODES AND SLOPES OF PANELS    C
C             COORDINATES ARE READ FROM INPUT DATA FILE UNLESS         C
C                  THE AIRFOIL IS OF NACA XXXX OR NACA 230XX TYPE      C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     NACA(I)   .... DUMMY NACA NUMBER FOR TRANSFER TO SUBROUTINES
C                    BODY AND NACA
C     TAU(I)    .... DUMMY MAX THICKNESS FOR TRANSFER TO SUBROUTINES
C                    BODY AND NACA
C     EPSMAX(I) .... DUMMY MAX CAMBER FOR TRANSFER TO SUBROUTINES
C                    BODY AND NACA
C     PTMAX(I)  ....  DUMMY CHORDWISE POSN FOR TRANSFER TO SUBROUTINES
C                    BODY AND NACA
C     _________________________________________________________________
      SUBROUTINE SETUP
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +        COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +        NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +        COSTHL(201),SINTHL(201)
      COMMON /PARD/ NACAD(2),TAUD(2),EPSMAD(2),PTMAXD(2)
      COMMON /PAR/ NACA,TAU,EPSMAX,PTMAX
      COMMON /NUM/ PI,PI2INV
      PI      = 3.1415926585
      PI2INV  = .5/PI
C
C     SET COORDINATES OF NODES ON BODY SURFACE
C
      DO 210 I=1,NAIRFO
      NODTOT  = NLOWER + NUPPER
      K=(I-1)*(NODTOT+1)
      IF (IFLAG .NE. 0) GO TO 10
C
C     NACA SERIES AIRFOIL CALCULATIONS
C
      NACA=NACAD(I)
      TAU=TAUD(I)
      EPSMAX=EPSMAD(I)
      PTMAX=PTMAXD(I)
      NPOINT  = NLOWER
      SIGN    = -1.0
      NSTART  = 0
      DO 110  NSURF = 1,2
      DO 100  N = 1,NPOINT
      FRACT   = FLOAT(N-1)/FLOAT(NPOINT)
      Z       = .5*(1. - COS(PI*FRACT))
      J       = NSTART + N
      CALL BODY(Z,SIGN,XD,YD)
      XI(J+K) = XD
      YI(J+K) = YD
 100  CONTINUE
      NPOINT  = NUPPER
      SIGN    = 1.0
      NSTART  = NLOWER
 110  CONTINUE
      XI(I*(NODTOT+1)) = XI(1+K)
 120  YI(I*(NODTOT+1)) = YI(1+K)
      GO TO 210
C
C     AIRFOIL DATA INPUT THAT ARE NOT NACA SERIES AIRFOIL.
C
  10  READ (1,501) (XI(J+K),J=1,NODTOT+1)
      READ (1,501) (YI(J+K),J=1,NODTOT+1)
      WRITE (6,501) (XI(J+K),J=1,NODTOT+1)
      WRITE (6,501) (YI(J+K),J=1,NODTOT+1)
 501  FORMAT (6F10.6)
 210  CONTINUE
      NP1     = NODTOT + 1
      NP2     = NAIRFO*NP1+1
C
C     SET SLOPES OF PANELS AND CALCULATE AIRFOIL PERIMETER
C
      DO 220  I = 1,NAIRFO
      K       = (I-1)*(NODTOT+1)
      SS(I)   = 0.0
      DO 200  J = 1,NODTOT
      DX      = XI(J+1+K) - XI(J+K)
      DY      = YI(J+1+K) - YI(J+K)
      DIST    = SQRT(DX*DX +DY*DY)
      SS(I)   = SS(I) + DIST
      SINTHL(J+K) = DY/DIST
      COSTHL(J+K) = DX/DIST
 200  CONTINUE
 220  CONTINUE
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                       C
C     SUBROUTINE BODY(Z,SIGN,X,Y)                                       C
C                                                                       C
C             RETURN COORDINATES OF POINT ON THE BODY SURFACE           C
C                                                                       C
C                  Z = NODE-SPACING PARAMETER                           C
C                  X,Y = CARTESIAN COORDINATES                          C
C                  SIGN = +1.  FOR UPPER SURFACE                        C
C                         -1.  FOR LOWER SURFACE                        C
C                                                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE BODY(Z,SIGN,XD,YD)
      COMMON /PAR/ NACA,TAU,EPSMAX,PTMAX
      IF (SIGN .LT. 0.0)    Z = 1. - Z
      CALL NACA45(Z,THICK,CAMBER,BETA)
      XD      = Z - SIGN*THICK*SIN(BETA)
      YD      = CAMBER + SIGN*THICK*COS(BETA)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                       C
C     SUBROUTINE NACA45(Z,THICK,CAMBER,BETA)                            C
C                                                                       C
C             EVALUATE THICKNESS AND CAMBER                             C
C             FOR NACA 4- OR 5-DIGIT AIRFOIL                            C
C                                                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     Z      ...... ABSCISSA OF CAMBERLINE POINT
C     X,Y    ...... LOCAL CARTESIAN COORDINATES OF NACA-AIRFOIL
C     SIGN   ...... SURFACE INDICATOR: SIGN = 1, UPPER SURFACE
C                                         =-1, LOWER SURFACE
C     THICK  ...... THICKNESS AT THE POSITION Z
C     CAMBER ...... CAMBER AT THE POSITION Z
C     BETA   ...... ANGLE BETWEEN CAMBER LINE AND X-AXIS AT POSITION Z
C     TAU    ...... MAXIMUM THICKNESS (INPUT)
C     EPSMAX ...... MAXIMUM CAMBER (INPUT)
C     PTMAX  ...... COORDINATE POSITION OF MAXIMUM CAMBER (INPUT)
C     _________________________________________________________________
      SUBROUTINE NACA45(Z,THICK,CAMBER,BETA)
      COMMON /PAR/ NACA,TAU,EPSMAX,PTMAX
      THICK   = 0.0
      IF (Z .LT. 1.E-10)     GO TO 100
      THICK   = 5.*TAU*(.2969*SQRT(Z) - Z*(.126 + Z*(.3537
     +          - Z*(.2843 - Z*.1015))))
 100  IF (EPSMAX .EQ. 0.0) GO TO 130
```

```
      IF (NACA .GT. 9999) GO TO 140
C
C     CAMBERLINE OF NACA 4-DIGIT SERIES
C
      IF (Z .GT. PTMAX) GO TO 110
C
C     FORWARD PART OF CAMBER LINE
C
      CAMBER  = EPSMAX/PTMAX/PTMAX*(2.*PTMAX - Z)*Z
      DCAMDX  = 2.*EPSMAX/PTMAX/PTMAX*(PTMAX - Z)
      GO TO 120
C
C     AFT PART OF CAMBERLINE
C
 110  CAMBER  = EPSMAX/(1.-PTMAX)**2*(1. + Z - 2.*PTMAX)*(1. - Z)
      DCAMDX  = 2.*EPSMAX/(1.-PTMAX)**2*(PTMAX - Z)
 120  BETA    = ATAN(DCAMDX)
      RETURN
 130  CAMBER  = 0.0
      BETA    = 0.0
      RETURN
C
C     CAMBERLINE OF NACA 5-DIGIT SERIES
C
 140  IF (Z .GT. PTMAX)      GO TO 150
C
C     FORWARD PART OF CAMBER LINE
C
      W       = Z/PTMAX
      CAMBER  = EPSMAX*W*((W - 3.)*W + 3. - PTMAX)
      DCAMDX  = EPSMAX*3.*W*(1. - W)/PTMAX
      GO TO 120
C
C     AFT PART OF CAMBERLINE
C
 150  CAMBER  = EPSMAX*(1. - Z)
      DCAMDX  = - EPSMAX
      GO TO 120
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                       C
C     SUBROUTINE GAUSS(NRHS,M,NITR)                                     C
C                                                                       C
C        SOLUTION OF LINEAR ALGEBRAIC SYSTEM BY                         C
C        GAUSS ELIMINATION WITHOUT PARTIAL PIVOTING                     C
C                                                                       C
C             °A        = COEFFICIENT MATRIX                            C
C             NEQNS     = NUMBER OF EQUATIONS                           C
C             NRHS      = NUMBER OF RIGHT HAND SIDES                    C
C                                                                       C
C             RIGHT-HAND SIDES AND SOLUTIONS STORED IN                  C
C             COLUMNS NEQNS+1 THRU NEQNS+NRHS OF °A                     C
C                                                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE GAUSS(NRHS,M,NITR)
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +             COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +             NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +             COSTHL(201),SINTHL(201)
      COMMON /COF/ A(201,211),KEQNS
C     WRITE (9,900) M,NITR
C900  FORMAT (1X,'M =',I3,'     NITR =',I3/)
C     WRITE (9,910)(I,A(I,NP3),A(I,NP4),A(I,NP5),I=1,2*NODTOT)
 910  FORMAT (1X,I5,3E14.6)
      IF (M.EQ.0)  KEQNS = NODTOT
      NEQNS   = KEQNS * NAIRFO
      NP      = NEQNS + 1
      NTOT    = NEQNS + NRHS
      IF  (NITR .GT. 0) GO TO 160
C
C              GAUSS REDUCTION
C
      DO 150  I = 2,NEQNS
      IM      = I - 1
C
C              ELIMINATE (I-1)TH UNKNOWN FROM
C              ITH THRU (NEQNS)TH EQUATIONS
C
      DO 150  J = I,NEQNS
      R       = A(J,IM)/A(IM,IM)
      DO 150  K = I,NTOT
 150  A(J,K)  = A(J,K) - R*A(IM,K)
      GO TO 170
C
C              GAUSSIAN ELIMINATION ON ONLY THE RIGHT-HAND-SIDES
C
 160  DO 180  I = 2,NEQNS
      IM      = I - 1
      DO 180  J = I,NEQNS
      R       = A(J,IM)/A(IM,IM)
      DO 180  K = NP,NTOT
 180  A(J,K)  = A(J,K) - R*A(IM,K)
 170  CONTINUE
C
C              BACK SUBSTITUTION
C
      DO 220  K = NP,NTOT
      A(NEQNS,K) = A(NEQNS,K)/A(NEQNS,NEQNS)
      DO 210  L = 2,NEQNS
      I       = NEQNS + 1 - L
      IP      = I + 1
      DO 200  J = IP,NEQNS
 200  A(I,K)  = A(I,K) - A(I,J)*A(J,K)
 210  A(I,K)  = A(I,K)/A(I,I)
 220  CONTINUE
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     SUBROUTINE VELDIS(SINALF,COSALF)                                 C
C                                                                      C
C     COMPUTE STEADY FLOW PRESSURE DISTRIBUTION AND VELOCITY           C
```

```
C     POTENTIAL AT MID-POINTS OF PANELS FOR THE STEADY FLOW CASE       C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     VTANG ..... TANGENTIAL VELO COMP OF A FIXED POINT OF THE MOVING
C                 LOCAL FRAME OF REFERENCE.
C     PHI(I) ..... DISTURBANCE VELO POTENTIAL AT THE MID POINT OF THE
C                 I-TH PANEL
C     PHILE(L).... DIFFERENCE OF THE POTENTIALS OF THE LEADING EDGE TO
C                 THE LOWER TRAILING EDGE FOR THE RESPECTIVE AIRFOIL
C     PIN(L)  .... DIFFERENCE OF THE POTENTIALS AT A POINT 1000 CHORD
C                 LENGTH UPSTREAM OF THE LE FOR THE RESPECTIVE AIRFOIL
C     SUMC(L) .... GAMMA ASSOCIATED WITH THE INTEGRATION OF THE DISTURB-
C                 ANCE VELOCITY AROUND THE WHOLE AIRFOIL
C     __________________________________________________________________
      SUBROUTINE VELDIS
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +             COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +             NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +             COSTHL(201),SINTHL(201)
      COMMON /COF/ A(201,211),KEQNS
      COMMON /CPD/ CP(200),SCL,T,SCM,SGAM
      COMMON /NUM/ PI,PI2INV
      COMMON /SING/ Q(200),GAMMA(2),QK(200),GAMK(2)
      COMMON /POT/ PHI(200),PHIK(200)
      COMMON /GUST/ UG(200),VG(200),XGF,UGUST,VGUST
      COMMON /EXTV/ UE(200),VN(200)
      COMMON /GEOM/ SINALF(2),COSALF(2),OMEGA(2),UX(2),UY(2),PIVOT,
     +              XPRM,YPRM
      DIMENSION CKUTTA(2),PHITEL(2),PHITEU(2)
 7770 DIMENSION CONTR2(2)
 6666 REAL *8 PIN(2),VELX
      DIMENSION WGHT(5),PLOC(5),SUMC(2),
     +AANP1(50,50,5),AANP2(50,50,5),BBNP1(50,50,5),BBNP2(50,50,5),
     +COSTHP(102,6),SINTHP(102,6),AANP4(2),PHIL(102),UGU(100,6),
     +VGU(100,6),PHILE(2)
C
C     LOCATION AND WEIGHTING  VALVES FOR THE GAUSSIAN QUADRATURE USED
C     TO INTEGRATE FOR THE VELO POTENTIAL AROUND THE AIRFOIL
C
      DATA WGHT/.11846344,.23931434,.28444444,
     +          .23931434,.11846344/
      DATA PLOC/.04691008,.23076535,.50000000,
     +          .76923466,.95300899/
C
C     FIND VT AND CP AT MID-POINT OF I-TH PANEL
C
      DO 140  L=  1,NAIRFO
      KI      = (L-1)*NP1
      LI      = (L-1)*NODTOT
      SUMC(L) = 0.0
      DO 130  I = 1,NODTOT
      XMID    = .5*(X(I+KI) +X(I+KI+1))
      YMID    = .5*(Y(I+KI) +Y(I+KI+1))
      DX      = X(I+KI+1)-X(I+KI)
      DY      = Y(I+KI+1)-Y(I+KI)
      DIST    = SQRT(DX*DX+DY*DY)
      VTANG   = COSALF(L)*COSTHL(I+KI) + SINALF(L)*SINTHL(I+KI)
      VNORM   = SINALF(L)*COSTHL(I+KI) - COSALF(L)*SINTHL(I+KI)
      VTFREE  = VTANG
      VACT    = VTANG
C
C     ADD CONTRIBUTION OF J-TH PANEL
C     1.  CONTRIBUTION OF J-TH PANEL TO THE VELO COMP OF THE MIDPT
C     OF THE I-TH PANEL
C
C     2.  CONTRIBUTION TO THE VELO POTENTIAL. THIS IS DONE BY
C     INTEGRATING OVER SMALLER PANELS OF THE AIRFOIL.
C
      DO 155  K = 1,5
      DX      = PLOC(K)*(X(I+KI+1)-X(I+KI))
      DY      = PLOC(K)*(Y(I+KI+1)-Y(I+KI))
      XMID    = X(I+KI) + DX
      YMID    = Y(I+KI) + DY
      VDUM    = 0.0
      DO 150  LM = 1,NAIRFO
      KJ      = (LM-1)*NP1
      LJ      = (LM-1)*NODTOT
      DO 120  J = 1,NODTOT
      FLOG    = 0.0
      FTAN    = PI
      IF (J+KJ .EQ. I+KI) GO TO 100
      DXJ     = XMID - X(J+KJ)
      DXJP    = XMID - X(J+KJ+1)
      DYJ     = YMID - Y(J+KJ)
      DYJP    = YMID - Y(J+KJ+1)
      FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
 100  CTIMTJ  = COSTHE(I+KI)*COSTHE(J+KJ) + SINTHE(I+KI)*SINTHE(J+KJ)
      STIMTJ  = SINTHE(I+KI)*COSTHE(J+KJ) - COSTHE(I+KI)*SINTHE(J+KJ)
      AA      = PI2INV*(FTAN*CTIMTJ + FLOG*STIMTJ)
      B       = PI2INV*(FLOG*CTIMTJ - FTAN*STIMTJ)
      VDUM    = VDUM - B*Q(J+LJ) +GAMMA(LM)*AA
      IF(K .EQ. 3) VACT = VACT - B*Q(J+LJ) + GAMMA(LM)*AA
      IF(K .EQ. 3) VNORM = VNORM + AA*Q(J+LJ) + GAMMA(LM)*B
 120  CONTINUE
 150  CONTINUE
      VTANG   = VTANG +VDUM *WGHT(K)
      SUMC(L) = SUMC(L) +VDUM*DIST*WGHT(K)
 155  CONTINUE
      PHI(I+LI) = (VTANG-VTFREE)*DIST
      CP(I+LI)= 1.0 - VACT*VACT
      UE(I+LI)= VACT
      VN(I+LI)= VNORM
 130  CONTINUE
 140  CONTINUE
C
C     COMPUTE DISTURBANCE POTENTIAL AT THE LEADING EDGE BY LINE
C                 INTEGRAL OF THE VELOCITY FIELD
C         FROM UPSTREAM (AT INFINITY) TO THE LEADING EDGE
C
      DO 55    L = 1,NAIRFO
      YMID    = PIVOT * SINALF(L) + (L-1)*YSHIFT
      XLE     = -PIVOT*COSALF(L)+(L-1)*XSHIFT
      XL      = XLE
C     XL      = 0.0
      NPHI    = 10 * NLOWER
      PIN(L)  = 0.0
      DO 30   I = 1,NPHI
      FRACT   = FLOAT(I)/FLOAT(NPHI)
      XLP     = -100.0 * (1.0 - COS(0.5*PI*FRACT))+XLE
      IF (I .EQ. 1) XLP = -0.000197+XLE
C     XLP     = -10.0 * (1.0 - COS(0.5*PI*FRACT))
      DELX    = XL - XLP
      XMID    = 0.5*(XL+XLP)
C     XMID    = 0.5*(XL+XLP)*COSALF(L)
C     YMID    = 0.5*(XL+XLP)*SINALF(L)
      XL      = XLP
      VELX    =  UGUST
C
C     ADD CONTRIBUTION OF J-TH PANEL
C
      DO 40   LN = 1,NAIRFO
      KJ      = (LN-1)*NP1
      LJ      = (LN-1)*NODTOT
      DO 20   J = 1,NODTOT
      DXJ     = XMID - X(J+KJ)
      DXJP    = XMID - X(J+KJ+1)
      DYJ     = YMID - Y(J+KJ)
      DYJP    = YMID - Y(J+KJ+1)
      FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
      CALMTJ  = -COSALF(L)*COSTHE(J+KJ) - SINALF(L)*SINTHE(J+KJ)
      SALMTJ  = -SINALF(L)*COSTHE(J+KJ) + COSALF(L)*SINTHE(J+KJ)
      APY     = PI2INV*(FTAN*CALMTJ + FLOG*SALMTJ)
      BPY     = PI2INV*(FLOG*CALMTJ - FTAN*SALMTJ)
      VELX    = VELX  - DPROD(BPY,Q(J+LJ)) +DPROD(GAMMA(LN),APY)
 20   CONTINUE
 40   CONTINUE
      PIN(L)  = PIN(L) + VELX * DBLE(DELX)
 30   CONTINUE
 55   CONTINUE
C
C     COMPUTATION OF THE VELOCITY POTENTIAL FOR MIDPOINT OF EACH PANEL
C
      DO 240 L   = 1,NAIRFO
      LI         = (L-1)*NODTOT
      PHP        = -PIN(L)
C
C     BEGIN WITH LOWER SURFACE
C
      DO 230 I   = NLOWER,1,-1
      PHC        = PHP-PHI(I+LI)
      PHI(I+LI) = 0.5*(PHP+PHC)
 230  PHP        = PHC
      PHITEL(L) = PHC
C
```

```
C     RESET FOR UPPER SURFACE
C
      PHP       = -PIN(L)
      DO 250 I   = NLOWER+1,NODTOT
      PHC       = PHP+PHI(I+LI)
      PHI(I+LI) = 0.5*(PHP+PHC)
 250  PHP       = PHC
      PHITEU(L) = PHC
 240  CONTINUE
      DO 334 L = 1 , NAIRFO
      WRITE (6,1010) L
      WRITE (6,1000)
      KI       = (L-1) *  NP1
      LI       = (L-1) *  NODTOT
      SUMC(L)  = SUMC(L)/SS(L)
      DO 333 I = 1 , NODTOT
      XMID    = .5*(X(I+KI) + X(I+KI+1))
      YMID    = .5*(Y(I+KI) + Y(I+KI+1))
  333 WRITE (6,1050) I+LI,XMID,YMID,Q(I+LI),GAMMA(L),CP(I+LI),UE(I+LI),
     + VN(I+LI),PHI(I+LI),SUMC(L)
  334 WRITE (6,235) PIN(L)
 235  FORMAT (1X,'PHI AT LEADING EDGE =',F10.6,/)
 1000 FORMAT(/,4X,'J',4X,'X(J)',5X,'Y(J)',6X,'Q(J)',6X,'GAMMA',5X,
     + 'CP(J)',6X,'V(J)',6X,'VN(J)',6X,'PHI',4X,'INTGAMMA',/)
 1010 FORMAT(//' AIRFOIL NUMBER',I4,/)
 1050 FORMAT(I5,9F10.6)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     SUBROUTINE FANDM(SINALF,COSALF)                                  C
C                                                                      C
C             COMPUTE AND PRINT OUT CD,CL,CM                           C
C             INTEGRATE PRESSURE DISTRIBUTION BY TRAPEZOIDAL RULE      C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CP(I) ....... PRESSURE COEFFICIENT OF THE I-TH PANEL
C     CL(L) ....... COEFFICIENT OF LIFT FOR THE L-TH AIRFOIL
C     CD(L) ....... COEFFICIENT OF DRAG FOR THE L-TH AIRFOIL
C     CM(L) ....... COEFFICIENT OF MOMENT FOR THE L-TH AIRFOIL WITH
C                   RESPECT TO THE LEADING EDGE
C     CFX(L) ...... COEFFICEIENT OF TOTAL FORCE IN X-DIR OF GLOBAL SYS.
C     CFY(L) ...... COEFFICEIENT OF TOTAL FORCE IN Y-DIR OF GLOBAL SYS.
C     _________________________________________________________________
C
      SUBROUTINE FANDM(SIN1,SIN2,COS1,COS2)
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +             COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +             NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +             COSTHL(201),SINTHL(201)
      COMMON /CORV/ CV(2,200),XC(2,200),YC(2,200),M,TD,CVVX(2,200),
     +              CVVY(2,200),XCI(2,200),YCI(2,200)
      COMMON /CPD/ CP(200),SCL,T,SCM,SGAM
      COMMON /SING/ Q(200),GAMMA(2),QK(200),GAMK(2)
      COMMON /GEOM/ SINALF(2),COSALF(2),OMEGA(2),UX(2),UY(2),PIVOT,
     +              XPRM,YPRM
      DIMENSION CM(2),CD(2),CL(2),CFX(2),CFY(2)
C
C     INITIALISE COEFFICIENT
C
      DO 110 L = 1,NAIRFO
      CM(L)   = 0.0
      CFX(L)  = 0.0
 110  CFY(L)  = 0.0
C
C     INTEGRATE AROUND THE AIRFOIL TO GET GLOBAL X AND Y FORCES
C
      DO 120  L = 1,NAIRFO
      KI      = (L-1)*(NODTOT+1)
      LI = (L-1)*NODTOT
      DO 100  I = 1,NODTOT
      XMID    = .5*(XI(I+KI) + XI(I+KI+1))
      YMID    = .5*(YI(I+KI) + YI(I+KI+1))
      DX      = XI(I+KI+1) - XI(I+KI)
      DY      = YI(I+KI+1) - YI(I+KI)
      CFX(L)  = CFX(L) + CP(I+LI)*DY
      CFY(L)  = CFY(L) - CP(I+LI)*DX
      CM(L)  = CM(L) + CP(I+LI)*(DX*XMID + DY*YMID)
 100  CONTINUE
 120  CONTINUE
C
C     DECOMPOSE INTO LIFT AND DRAG COMPONENTS W.R.T. RESP LOCAL SYSTEM
C
      DO 130 L =1,NAIRFO
      CD(L)   = CFX(L)*COSALF(L) + CFY(L)*SINALF(L)
      CL(L)   = CFY(L)*COSALF(L) - CFX(L)*SINALF(L)
 130  WRITE (6,1000) L,CD(L),CL(L),CM(L)
      IF (M .EQ. 0) RETURN
      IF (SCL .NE. 0.0) THEN
      WRITE (8,1100) T,CL(1)/SCL,CL(2)/SCL,CM(1)/SCM,CM(2)/SCM,CD(1),
     +CD(2),GAMK(1)/SGAM,GAMK(2)/SGAM
      ELSE
      WRITE (8,1100) T,CL(1),CL(2),CM(1),CM(2),CD(1),CD(2)
      ENDIF
 1000 FORMAT(//,' AIRFOIL NO ',I4,/,'    CD =',F10.6,
     +           '    CL=',F10.6,'    CM=',F10.6)
 1100 FORMAT(9F10.6)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     SUBROUTINE INFL (NITR)                                           C
C                                                                      C
C             CALCULATE INFLUENCE COEFFICIENTS                         C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     INFLUENCE COEFFICIENTS ON THE AIRFOIL DUE TO THE AIRFOIL:
C     AAN(I,J)  .... NORMAL VELO AT THE MIDPOINT OF THE I-TH PANEL DUE
C                    TO A SOURCE-DIST OF UNIT STRENGTH ON THE J-TH PANEL
C     SUMAAN(I,L) .. NORMAL VELO AT THE MIDPOINT OF THE I-TH PANEL DUE
```

```
C                    SOURCE DIST OF UNIT STRENGTH FROM THE L AIRFOIL
C     BBN(I,J)  .... NORMAL VELO AT THE MIDPOINT OF THE I-TH PANEL DUE
C                    TO A VORTEX-DIST OF UNIT STRENGTH ON THE J-TH PANEL
C     SUMAAN(I,L) .. NORMAL VELO AT THE MIDPOINT OF THE I-TH PANEL DUE
C                    VORTEX DIST OF UNIT STRENGTH FROM THE L AIRFOIL
C
C     INFLUENCE COEFFICIENT ON THE WAKE ELEMENT:
C     AYNP1(L,J) ... Y-VELO COMP AT THE MIDPOINT OF THE WAKE PANEL FROM
C                    THE L-TH AIRFOIL DUE TO A SOURCE DIST OF UNIT STRE-
C                    NGTH FROM THE J-TH PANEL
C     AYNP1(L,NP3) . Y-VELO COMP AT THE MIDPOINT OF THE WAKE PANEL FROM
C                    THE L-TH PANEL DUE TO A SOURCE DIST OF UNIT STREN-
C                    GTH FROM THE WAKE PANEL OF THE OTHER AIRFOIL.
C                    (USED ONLY FOR BXNP1(L,NP3) SINCE THERE IS NO SOURC
C                    DIST ON THE WAKE PANEL)
C     BYNP1(L,J) ... Y-VELO COMP AT THE MIDPOINT OF THE WAKE PANEL FROM
C                    THE L-TH AIRFOIL DUE TO A VORTEX DIST OF UNIT STRE-
C                    NGTH FROM THE J-TH PANEL
C     BYNP1(L,NP3) . Y-VELO COMP AT THE MIDPOINT OF THE WAKE PANEL FROM
C                    THE L-TH AIRFOIL DUE TO A VORTEX DIST OF UNIT STR-
C                    ENGTH FROM THE WAKE PANEL OF THE OTHER AIRFOIL.
C     CYNP1(L,N) ... Y-VELO COMP AT THE MIDPOINT OF THE WAKE PANEL FROM
C                    THE L-TH AIRFOIL DUE TO THE N-TH CORE VORTEX OF
C                    UNIT STRENGTH
C     CXNP1(L,N) ... X-VELO COMP AT THE MIDPOINT OF THE WAKE PANEL FROM
C                    THE L-TH AIRFOIL DUE TO THE N-TH CORE VORTEX OF
C                    UNIT STRENGTH
C
C     INFLUENCE COEFFICIENTS ON THE AIRFOIL DUE TO THE WAKE
C     SUMCCN(I) ... NORMAL VELO AT THE MIDPOINT OF THE I-TH PANEL DUE
C                   TO ALL POINT VORTICES OF ACTUAL STRENGTH.
C     SUMCCT(I) ... TANGENTIAL VELO AT THE MIDPOINT OF THE I-TH PANEL
C                   DUE TO ALL POINT VORTICES OF ACTUAL STRENGTH.
C
C
C
      SUBROUTINE INFL (NITR)
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +             COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +             NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +             COSTHL(201),SINTHL(201)
      COMMON /NUM/ PI,PI2INV
      COMMON /WAK/ VYW(2),VXW(2),WAKE(2),DT
      COMMON /CORV/ CV(2,200),XC(2,200),YC(2,200),M,TD,CVVX(2,200),
     +              CVVY(2,200),XCI(2,200),YCI(2,200)
      COMMON /INF1/ AAN(201,201),BBN(201,201),AYNP1(2,201),BYNP1(2,201),
     +              SUMAAN(201,2),SUMBBN(201,2)
      COMMON /INF2/ SUMCCN(201),SUMCCT(201),CYNP1(2,200),
     +              CXNP1(2,200)
      COMMON /GEOM/ SINALF(2),COSALF(2),OMEGA(2),UX(2),UY(2),PIVOT,
     +              XPRM,YPRM
      DIMENSION GAYNP1(2,201),GBYNP1(2,201)
      IF (M .GT. 1)  GO TO 510
C
C     INFLUENCE COEFFICIENT ON THE I-TH PANEL BY THE J-TH PANEL FROM
C     THE SAME AIRFOIL.
C
```

```
      DO 200  L = 1,NAIRFO
      LI      = (L-1)*NODTOT
      KI      = (L-1)*NP1
      DO 120  I = 1,NODTOT
      XMID    = .5*(XI(I+KI) + XI(I+KI+1))
      YMID    = .5*(YI(I+KI) + YI(I+KI+1))
      SUMAAN(I+LI,L) = 0.0
      SUMBBN(I+LI,L) = 0.0
      DO 110  J = 1,NODTOT
      FLOG    = 0.0
      FTAN    = PI
      IF (J+LI .EQ. I+LI)     GO TO 100
      DXJ     = XMID - XI(J+KI)
      DXJP    = XMID - XI(J+KI+1)
      DYJ     = YMID - YI(J+KI)
      DYJP    = YMID - YI(J+KI+1)
      FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
  100 CTIMTJ  = COSTHL(I+KI)*COSTHL(J+KI) + SINTHL(I+KI)*SINTHL(J+KI)
      STIMTJ  = SINTHL(I+KI)*COSTHL(J+KI) - COSTHL(I+KI)*SINTHL(J+KI)
      AAN(I+LI,J+LI) = PI2INV*(FTAN*CTIMTJ + FLOG*STIMTJ)
      BBN(I+LI,J+LI) = PI2INV*(FLOG*CTIMTJ - FTAN*STIMTJ)
      SUMAAN(I+LI,L) = SUMAAN(I+LI,L) + AAN(I+LI,J+LI)
      SUMBBN(I+LI,L) = SUMBBN(I+LI,L) + BBN(I+LI,J+LI)
  110 CONTINUE
  120 CONTINUE
  200 CONTINUE
  510 CONTINUE
      IF (NITR .GT. 0) GO TO 271
C
C     INFLUENCE COEFFICIENT ON THE I-TH PANEL BY THE J-TH PANEL FROM
C     THE OTHER AIRFOIL.
C
      DO 270  L = 1,NAIRFO
      LI      = (L-1)*NODTOT
      KI      = (L-1)*NP1
      DO 260  I = 1,NODTOT
      XMID    = .5*(X(I+KI) + X(I+KI+1))
      YMID    = .5*(Y(I+KI) + Y(I+KI+1))
      SUMAAN(I+LI,3-L) = 0.0
      SUMBBN(I+LI,3-L) = 0.0
      DO 250  J = NODTOT+2,2*NODTOT+1
      DXJ     = XMID - X(J-KI)
      DXJP    = XMID - X(J-KI+1)
      DYJ     = YMID - Y(J-KI)
      DYJP    = YMID - Y(J-KI+1)
      FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
      CTIMTJ  = COSTHE(I+KI)*COSTHE(J-KI) + SINTHE(I+KI)*SINTHE(J-KI)
      STIMTJ  = SINTHE(I+KI)*COSTHE(J-KI) - COSTHE(I+KI)*SINTHE(J-KI)
      AAN(I+LI,J-LI-1) = PI2INV*(FTAN*CTIMTJ + FLOG*STIMTJ)
      BBN(I+LI,J-LI-1) = PI2INV*(FLOG*CTIMTJ - FTAN*STIMTJ)
      SUMAAN(I+LI,3-L) = SUMAAN(I+LI,3-L) + AAN(I+LI,J-LI-1)
      SUMBBN(I+LI,3-L) = SUMBBN(I+LI,3-L) + BBN(I+LI,J-LI-1)
  250 CONTINUE
  260 CONTINUE
  270 CONTINUE
C
C     END OF STEADY FLOW CALAULATION FOR INFLUENCE COEFFICIENT.
C
      IF (M.EQ.0)  RETURN
  271 CONTINUE
C
C     INFLUENCE COEFFICIENT ON THE WAKE ELEMENT FROM THE AIRFOIL.
C
      DO 130  L = 1,NAIRFO
      I       = NP1*L
      XMID    = .5*(X(I) + X(NAIRFO*NP1+L))
      YMID    = .5*(Y(I) + Y(NAIRFO*NP1+L))
      DO 130  MM = 1,NAIRFO
      LJ      =(MM-1)*NODTOT
      KJ      =(MM-1)*NP1
      DO 130  J = 1,NODTOT
      DXJ     = XMID - X(J+KJ)
      DXJP    = XMID - X(J+KJ+1)
      DYJ     = YMID - Y(J+KJ)
      DYJP    = YMID - Y(J+KJ+1)
      FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
      CTIMTJ  = COSTHE(I)*COSTHE(J+KJ) + SINTHE(I)*SINTHE(J+KJ)
      STIMTJ  = SINTHE(I)*COSTHE(J+KJ) - COSTHE(I)*SINTHE(J+KJ)
      GAYNP1(L,J+LJ) = PI2INV*(FTAN*COSTHE(J+KJ) - FLOG*SINTHE(J+KJ))
      GBYNP1(L,J+LJ) = PI2INV*(FLOG*COSTHE(J+KJ) + FTAN*SINTHE(J+KJ))
      AYNP1(L,J+LJ) = GAYNP1(L,J+LJ)*COSALF(L)-GBYNP1(L,J+LJ)*SINALF(L)
      BYNP1(L,J+LJ) = GAYNP1(L,J+LJ)*SINALF(L)+GBYNP1(L,J+LJ)*COSALF(L)
  130 CONTINUE
C
C     INFLUENCE COEFICIENT OF WAKE ELEMENT DUE TO WAKE ELEMENT FROM
C     THE OTHER AIRFOIL
C
      DO 300  L = 1,NAIRFO
      I       = NP1*L
      XMID    = .5*(X(I) + X(NAIRFO*NP1+L))
      YMID    = .5*(Y(I) + Y(NAIRFO*NP1+L))
      KJ      = (L-1)*NP1
      MJ      = 2*NP1
      NJ      = MJ+3
      DXJ     = XMID - X(MJ-KJ)
      DXJP    = XMID - X(NJ-L)
      DYJ     = YMID - Y(MJ-KJ)
      DYJP    = YMID - Y(NJ-L)
      FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
      CTIMTJ  = COSTHE(I)*COSTHE(MJ-KJ) + SINTHE(I)*SINTHE(MJ-KJ)
      STIMTJ  = SINTHE(I)*COSTHE(MJ-KJ) - COSTHE(I)*SINTHE(MJ-KJ)
      GAYNP1(L,NP3) = PI2INV*(FTAN*COSTHE(MJ-KJ) - FLOG*SINTHE(MJ-KJ))
      GBYNP1(L,NP3) = PI2INV*(FLOG*COSTHE(MJ-KJ) + FTAN*SINTHE(MJ-KJ))
      AYNP1(L,NP3)  = GAYNP1(L,NP3)*COSALF(L)-GBYNP1(L,NP3)*SINALF(L)
      BYNP1(L,NP3)  = GAYNP1(L,NP3)*SINALF(L)+GBYNP1(L,NP3)*COSALF(L)
  300 CONTINUE
C
```

```
C     INFLUENCE COEFFICIENT ON THE AIRFOIL BY THE WAKE ELEMENT.
C
      DO 160  L = 1,NAIRFO
      LI=(L-1)*NODTOT
      KI=(L-1)*NP1
      DO 140  I = 1,NODTOT
      XMID    = .5*(X(I+KI) + X(I+KI+1))
      YMID    = .5*(Y(I+KI) + Y(I+KI+1))
      DO 145  MM = 1, NAIRFO
      J       = NP1*MM
      DXJ     = XMID - X(J)
      DXJP    = XMID - X(NAIRFO*NP1+MM)
      DYJ     = YMID - Y(J)
      DYJP    = YMID - Y(NAIRFO*NP1+MM)
      FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
      CTIMTJ  = COSTHE(I+KI)*COSTHE(J) + SINTHE(I+KI)*SINTHE(J)
      STIMTJ  = SINTHE(I+KI)*COSTHE(J) - COSTHE(I+KI)*SINTHE(J)
      AAN(I+LI,2*NODTOT+MM) = PI2INV*(FTAN*CTIMTJ + FLOG*STIMTJ)
      BBN(I+LI,2*NODTOT+MM) = PI2INV*(FLOG*CTIMTJ - FTAN*STIMTJ)
 145  CONTINUE
 140  CONTINUE
 160  CONTINUE
C
      IF (M.EQ.1) RETURN
      MM1     = M - 1
C
C     INFLUENCE COEFFICIENT ON THE WAKE ELEMENT BY THE CORE VORTICES.
C
      DO 350  L = 1,NAIRFO

      XMID    = 0.5*(X(NP1*L) + X(NP1*NAIRFO+L))
      YMID    = 0.5*(Y(NP1*L) + Y(NP1*NAIRFO+L))
      DO 240  MM = 1, NAIRFO
      KN      = (MM-1)*MM1
      DO 230  N = 1,MM1
      DX      = XMID - XC(MM,N)
      DY      = YMID - YC(MM,N)
      DIST2   = DX*DX+DY*DY
      GCYNP1  = -PI2INV*DX/DIST2
      GCXNP1  = +PI2INV*DY/DIST2
      CYNP1(L,N+KN) = GCYNP1*COSALF(L)+GCXNP1*SINALF(L)
      CXNP1(L,N+KN) = -GCYNP1*SINALF(L)+GCXNP1*COSALF(L)
 230  CONTINUE
 240  CONTINUE
 350  CONTINUE
      IF (NITR.GT.0) RETURN
C
C     INFLUENCE COEFFICIENT ON THE AIRFOIL BY THE CORE VORTICES
C
      DO 400  L = 1,NAIRFO
      LI      =(L-1)*NODTOT
      KI      =(L-1)*NP1
      DO 220  I = 1,NODTOT
      XMID    = 0.5*(X(I+KI) + X(I+KI+1))
      YMID    = 0.5*(Y(I+KI) + Y(I+KI+1))
      SUMCCN(I+LI) = 0.0
      SUMCCT(I+LI) = 0.0
      DO 280 MM = 1, NAIRFO
      KN       = (MM-1)*MM1
      DO 210  N = 1,MM1
      DX       = XMID - XC(MM,N)
      DY       = YMID - YC(MM,N)
      DIST     = SQRT(DX*DX+DY*DY)
      COSTHN   = DX/DIST
      SINTHN   = DY/DIST
      CTIMTN   = COSTHE(I+KI)*COSTHN + SINTHE(I+KI)*SINTHN
      STIMTN   = SINTHE(I+KI)*COSTHN - COSTHE(I+KI)*SINTHN
      CCN      = -CTIMTN/DIST
      CCT      = -STIMTN/DIST
      SUMCCN(I+LI) = SUMCCN(I+LI) + CCN*CV(MM,N)
      SUMCCT(I+LI) = SUMCCT(I+LI) + CCT*CV(MM,N)
 210  CONTINUE
 280  CONTINUE
      SUMCCN(I+LI) = PI2INV*SUMCCN(I+LI)
      SUMCCT(I+LI) = PI2INV*SUMCCT(I+LI)
 220  CONTINUE
 400  CONTINUE
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     SUBROUTINE COEF (SINALF,COSALF,OMEGA,UX,UY,NITR)                 C
C                                                                      C
C             SET COEFFICIENTS OF N EQUS ARISING FROM FLOW             C
C                  TANGENCY CONDITIONS AT MID POINTS OF PANELS         C
C             SOLVING THE N-SOURCE STRENGTHS IN TERMS OF THE           C
C                  VORTICITY STRENGTH (RESULTING IN 2 RHS)             C
C             KUTTA CONDITION IS SATISFIED SEPARATELY TO OBTAIN        C
C                  THE VORTICITY STRENGTH                              C
C             THIS SOLUTION METHOD IS DESIRED FOR UNSTEADY FLOW        C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     A(I,J)  ..... LHS, NORMAL VELOCITY AT THE MIDPOINT OF THE I-TH
C                   PANEL INDUCED BY UNIT SOURCE DIST ON THE J-TH PANEL
C     A(I,NP3) .... FIRST RHS, COMPONENT OF NORMAL VELO AT THE I-TH
C                   PANEL WHICH IS DEPENDENT ON VORTICITY STRENGTH OF
C                   THE FIRST AIRFOIL
C     A(I,NP4) .... SECOND RHS, COMPONENT OF NORMAL VELO AT THE I-TH
C                   PANEL WHICH IS DEPENDENT ON VORTICITY STRENGTH OF
C                   THE SECOND AIRFOIL
C     A(I,NP5) .... THIRD RHS, COMPONENT OF NORMAL VELO AT THE I-TH
C                   PANEL WHICH IS INDEPENDENT OF THE CIRCULATION OF
C                   BOTH AIRFOILS
C     _________________________________________________________________

      SUBROUTINE COEF (NITR)
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +             COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +             NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +             COSTHL(201),SINTHL(201)
```

```
      COMMON /COF/ A(201,211),KEQNS
      COMMON /SING/ Q(200),GAMMA(2),QK(200),GAMK(2)
      COMMON /WAK/ VYW(2),VXW(2),WAKE(2),DT
      COMMON /CORV/ CV(2,200),XC(2,200),YC(2,200),M,TD,CVVX(2,200),
     +              CVVY(2,200),XCI(2,200),YCI(2,200)
      COMMON /INF1/ AAN(201,201),BBN(201,201),AYNP1(2,201),BYNP1(2,201),
     +              SUMAAN(201,2),SUMBBN(201,2)
      COMMON /INF2/ SUMCCN(201),SUMCCT(201),CYNP1(2,200),
     +              CXNP1(2,200)
      COMMON /GUST/ UG(200),VG(200),XGF,UGUST,VGUST
      COMMON /MATRIX/ AMAT(201,211)
      COMMON /GEOM/ SINALF(2),COSALF(2),OMEGA(2),UX(2),UY(2),PIVOT,
     +              XPRM,YPRM
      NEQS    = NODTOT* NAIRFO
      IF (NITR .GT. 0) GO TO 91
C
C     SET/RESET LHS MATRIX A(I,J) FOR EACH NEW TIME STEP
C
      DO 110  I = 1,2*NODTOT
      DO 110  J = 1,2*NODTOT
 110  A(I,J) = AAN(I,J)
      IF (M.NE.0)  GOTO 91
C
C     FILL IN THE RIGHT HAND SIDE FOR STEADY FLOW
C
      DO 310 L=1,NAIRFO
      LI      = (L-1)*NODTOT
      KI      = (L-1)*NP1
      DO 310  I = 1,NODTOT
      XMID    = 0.5 * (XI(I+KI) + XI(I+KI+1))
      YMID    = 0.5 * (YI(I+KI) + YI(I+KI+1))
      A(I+LI,NP3) = -SUMBBN(I+LI,1)
      A(I+LI,NP4) = -SUMBBN(I+LI,2)
  310 A(I+LI,NP5) = SINTHL(I+KI)*COSALF(L) - COSTHL(I+KI)*SINALF(L)
      RETURN
C
C             FILL IN THE RIGHT HAND SIDE FOR UNSTEADY FLOW
C
   91 DO 210 L=1,NAIRFO
      LI      = (L-1)*NODTOT
      KI      = (L-1)*NP1
      DO 210  I = 1,NODTOT
      XMID    = 0.5 * (XI(I+KI) + XI(I+KI+1))
      YMID    = 0.5 * (YI(I+KI) + YI(I+KI+1))
      A(I+LI,NP3) = -SUMBBN(I+LI,1) + BBN(I+LI,NP3)*SS(1)/WAKE(1)
      A(I+LI,NP4) = -SUMBBN(I+LI,2) + BBN(I+LI,NP4)*SS(2)/WAKE(2)
      A(I+LI,NP5) = -BBN(I+LI,NP3)*GAMMA(1)*SS(1)/WAKE(1)-BBN(I+LI,NP4)
     +*GAMMA(2)*SS(2)/WAKE(2) + SINTHL(I+KI)* ((1.+UG(I+LI))*COSALF(L)-
     +VG(I+LI)*SINALF(L)+UX(L)) - COSTHL(I+KI)* ((1.+UG(I+LI))
     +*SINALF(L)+VG(I+LI)*COSALF(L)+UY(L))+OMEGA(L)*(YMID*SINTHL(I+KI)
     + + XMID*COSTHL(I+KI))
C
C         ADD CORE VORTEX CONTRIBUTION
C
      IF (M .EQ. 1) GOTO 210
      A(I+LI,NP5) = A(I+LI,NP5) - SUMCCN(I+LI)
 210  CONTINUE
      DO 300 I=1,2*NODTOT
      DO 300 J=1,2*NODTOT+3
  300 AMAT(I,J) = A(I,J)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     SUBROUTINE KUTTA (ALPHA,SINALF,COSALF,OMEGA,UX,UY)               C
C                                                                      C
C             USING KUTTA CONDITION TO DETERMINE VORTICITY             C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     GAMMA ... CIRCULATION FOR THE STEADY FLOW CASE AND ALSO
C               CIRCULATION FOR THE PREVIOUS TIME STEP FOR UNSTEADY CASE
C     GAMK  ... CIRCULATION AT THE CURRENT TIME STEP FOR UNSTEADY CASE
C     QK    ... SOURCE STRENGTH AT THE CURRENT TIME STEP
C               QK(I) = B1(I)*GAMK(1)+B2(I)*GAMK(2)+B3(I)
C     B1(I) ... THAT PART OF THE SOURCE STRNGTH WHICH IS INDUCED BY A
C               CIRCULATION OF UNIT STRENGTH FROM AIRFOIL 1
C     B2(I) ... THAT PART OF THE SOURCE STRNGTH WHICH IS INDUCED BY A
C               CIRCULATION OF UNIT STRENGTH FROM AIRFOIL 2
C     B3(I) ... THAT PART OF THE SOURCE STRNGTH WHICH IS INDEPENDENT
C               OF THE CIRCULATION FROM BOTH AIRFOILS
C     AA(I,J) . THAT PART OF THE TANGENTIAL VELOCITY AT THE TRAILING
C               EDGE PANELS WHICH IS INDUCED BY A CIRCULATION  OF UNIT
C               STRENGTH BY AIRFOIL J
C     BB(I) ... THAT PART OF THE TANGENTIAL VELOCITY AT THE TRAILING
C               EDGE PANELS WHICH IS INDEPENDENT OF THE CIRCULATION
C     _________________________________________________________________
      SUBROUTINE KUTTA(NITR,PVTAG)
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +             COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +             NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +             COSTHL(201),SINTHL(201)
      COMMON /COF/ A(201,211),KEQNS
      COMMON /SING/ Q(200),GAMMA(2),QK(200),GAMK(2)
      COMMON /WAK/ VYW(2),VXW(2),WAKE(2),DT
      COMMON /CORV/ CV(2,200),XC(2,200),YC(2,200),M,TD,CVVX(2,200),
     +              CVVY(2,200),XCI(2,200),YCI(2,200)
      COMMON /INF1/ AAN(201,201),BBN(201,201),AYNP1(2,201),BYNP1(2,201),
     +              SUMAAN(201,2),SUMBBN(201,2)
      COMMON /INF2/ SUMCCN(201),SUMCCT(201),CYNP1(2,200),
     +              CXNP1(2,200)
      COMMON /GUST/ UG(200),VG(200),XGF,UGUST,VGUST
      COMMON /MATRIX/ AMAT(201,211)
      COMMON /BMAT/ B1,B2,B3
      COMMON /GEOM/ SINALF(2),COSALF(2),OMEGA(2),UX(2),UY(2),PIVOT,
     +              XPRM,YPRM
      COMMON /GIES/ NGIES
      DIMENSION B1(200),B2(200),B3(200),AA(4,2),BB(5),EE(2),FF(2),
     +GG(2),RADI(2),AAA(2),BBB(2),CCC(2),DDD(2),EEE(2),FFF(2),
     +DG(2),E(2),GAMA(2),DELG(2),DGAMK(2,3),COEFL(2,3)
C
C     RETRIEVE SOLUTION FROM A-MATRIX FOR THE SOURCE STRENGTH IN TERMS
```

```
C     THE TWO GAMK CONTRIBUTION AND THE CONSTANT COMPONENT
C
      COUNT   = 0
      DO 50   I = 1,NODTOT*2
      B1(I)   = A(I,NP3)
      B2(I)   = A(I,NP4)
 50   B3(I)   = A(I,NP5)
      IF (M.GT.0)  GOTO 400
C
C     STEADY KUTTA CONDITION
C
C     COMPUTE TANGENTIAL VELOCITIES
      DO 425  L = 1,NAIRFO
      LI      = (L-1)*NODTOT
      KI      = (L-1)*NP1
      KK      = (L-1)*2
      DO 430  K = 1,2
      IF (K .EQ. 1) I = 1
      IF (K .EQ. 2) I = NODTOT
      XMID    = 0.5 * (XI(I+KI) + XI(I+KI+1))
      YMID    = 0.5 * (YI(I+KI) + YI(I+KI+1))
      AA(K+KK,1)  = SUMAAN(I+LI,1)
      AA(K+KK,2)  = SUMAAN(I+LI,2)
      BB(K+KK) =  COSALF(L)*COSTHL(I+KI) + SINALF(L)*SINTHL(I+KI)
      DO 419  J = 1,2*NODTOT
      AA(K+KK,1)   = AA(K+KK,1) - BBN(I+LI,J)*B1(J)
      AA(K+KK,2)   = AA(K+KK,2) - BBN(I+LI,J)*B2(J)
      BB(K+KK)   = BB(K+KK) - BBN(I+LI,J)*B3(J)
 419  CONTINUE
 430  CONTINUE
 425  CONTINUE
C
C     SET UP KUTTA CONDITIONS
C
      DO 450 I=1,2
      LI      = (I-1)*NAIRFO+1
      DGAMK(I,1) = AA(LI,1) + AA(LI+1,1)
      DGAMK(I,2) = AA(LI,2) + AA(LI+1,2)
  450 DGAMK(I,3) = -(BB(LI) + BB(LI+1))
C
C     SOLVE KUTTA CONDITION BY GAUSS
C
      R=DGAMK(2,1)/DGAMK(1,1)
      DO 460 K = 2,3
  460 DGAMK(2,K) = DGAMK(2,K)-R*DGAMK(1,K)
      GAMMA(2) = DGAMK(2,3)/DGAMK(2,2)
      GAMMA(1) = (DGAMK(1,3)-DGAMK(1,2)*GAMMA(2))/DGAMK(1,1)
C
C     CALCULATE SOURCE STRENGTH
C
      DO 470  L = 1,NAIRFO
      LI      =(L-1)*NODTOT
      DO 470  I = 1,NODTOT
 470  Q(I+LI)   = GAMMA(1)*B1(I+LI) + GAMMA(2)*B2(I+LI) + B3(I+LI)
      RETURN
C
```

```
C     UNSTEADY KUTTA CONDITION
C
C     FIND VT AT TRAILING EDGE PANELS
C
C     INITIAL GUESS FOR GAMK FROM PREVIOUS TIME STEP
 400  IF (NITR .EQ. 0) THEN
      GAMK(1) = GAMMA(1)
      GAMK(2) = GAMMA(2)
      ENDIF
      DO 125  L = 1,NAIRFO
      LI      = (L-1)*NODTOT
      KI      = (L-1)*NP1
      KK      = (L-1)*2
      DO 130  K = 1,2
      IF (K .EQ. 1) I = 1
      IF (K .EQ. 2) I = NODTOT
      XMID    = 0.5 * (XI(I+KI) + XI(I+KI+1))
      YMID    = 0.5 * (YI(I+KI) + YI(I+KI+1))
      VTANG   =  ((1.+UG(I+LI))*COSALF(L)-VG(I+LI)*SINALF(L)+UX(L))
     +*COSTHL(I+KI)+ ((1.+UG(I+LI))*SINALF(L)+VG(I+LI)*COSALF(L)+UY(L))
     +*SINTHL(I+KI) + OMEGA(L)*(YMID*COSTHL(I+KI)
     +- XMID*SINTHL(I+KI))
      AA(K+KK,1)  = - AAN(I+LI,NP3)*SS(1)/WAKE(1)
      AA(K+KK,2)  = - AAN(I+LI,NP4)*SS(2)/WAKE(2)
      BB(K+KK)  = VTANG + AAN(I+LI,NP3)*SS(1)*GAMMA(1)/WAKE(1) +
     +            AAN(I+LI,NP4)*SS(2)*GAMMA(2)/WAKE(2)
      DO 119  J = 1,NODTOT
      AA(K+KK,1)   = AA(K+KK,1) + AAN(I+LI,J) - BBN(I+LI,J)*B1(J)
      AA(K+KK,2)   = AA(K+KK,2) + AAN(I+LI,J+NODTOT) - BBN(I+LI,J)*B2(J)
      BB(K+KK)   = BB(K+KK) - BBN(I+LI,J)*B3(J)
 119  CONTINUE
      DO 120 JJ = NODTOT+1,2*NODTOT
      AA(K+KK,1)   = AA(K+KK,1) - BBN(I+LI,JJ)*B1(JJ)
      AA(K+KK,2)   = AA(K+KK,2) - BBN(I+LI,JJ)*B2(JJ)
      BB(K+KK)   = BB(K+KK) - BBN(I+LI,JJ)*B3(JJ)
 120  CONTINUE
C
C     ADD CORE VORTEX CONTRIBUTION
C
      IF (M.LE.1) GOTO 100
      BB(K+KK)   = BB(K+KK) + SUMCCT(I+LI)
 100  CONTINUE
 130  CONTINUE
 125  CONTINUE
      IF (NGIES .EQ. 1) GOTO 145
C
C     SATISFYING KUTTA CONDITION -- SOLVE FOR VORTEX STRENGTH
C
      DO 135 I = 1,2
      LI      = (I-1)*NAIRFO+1
      AAA(I)  = AA(LI,1)**2-AA(LI+1,1)**2
      BBB(I)  = AA(LI,2)**2-AA(LI+1,2)**2
      CCC(I)  = 2*(AA(LI,1)*BB(LI)-AA(LI+1,1)*BB(LI+1)-(2-I)*SS(1)/DT)
      DDD(I)  = 2*(AA(LI,2)*BB(LI)-AA(LI+1,2)*BB(LI+1)-(I-1)*SS(2)/DT)
      EEE(I)  = 2*(AA(LI,1)*AA(LI,2)-AA(LI+1,1)*AA(LI+1,2))
      FFF(I)  = BB(LI)**2-BB(LI+1)**2+2*SS(I)*GAMMA(I)/DT
 135  CONTINUE
  60  DO 140 I = 1,2
      DGAMK(I,1) = 2*AAA(I)*GAMK(1)+EEE(I)*GAMK(2)+CCC(I)
      DGAMK(I,2) = 2*BBB(I)*GAMK(2)+EEE(I)*GAMK(1)+DDD(I)
 140  DGAMK(I,3) = -(AAA(I)*GAMK(1)**2+CCC(I)*GAMK(1)+BBB(I)*
     + GAMK(2)**2+DDD(I)*GAMK(2)+EEE(I)*GAMK(1)*GAMK(2)+FFF(I))
C140  WRITE (6,*)'DGAMK',I,DGAMK(I,1),DGAMK(I,2),DGAMK(I,3)
      DO 144 I=1,2
      DO 144 KKK=1,3
 144  COEFL(I,KKK) = DGAMK(I,KKK)
C
C     GAUSSIAN ELIMINATION
C
      R=DGAMK(2,1)/DGAMK(1,1)
      DO 200 K = 2,3
 200  DGAMK(2,K) = DGAMK(2,K)-R*DGAMK(1,K)
      DG(2) = DGAMK(2,3)/DGAMK(2,2)
      DG(1) = (DGAMK(1,3)-DGAMK(1,2)*DG(2))/DGAMK(1,1)
      GAMK(1) = GAMK(1)+DG(1)
      GAMK(2) = GAMK(2)+DG(2)
      IF ((ABS(DG(1)/GAMK(1)) .LT..0001) .AND.
     +    (ABS(DG(2)/GAMK(2)).LT..0001))  GO TO 300
      COUNT = COUNT + 1
      IF (COUNT .EQ. 50) GO TO 310
      GO TO 60
 310  WRITE(6,*)'KUTTA CONDITION NOT SATISFIED -- COUNT EXCEEDED 50'
      RETURN
C
C  SET UP KUTTA CONDITIONS FOR GIESING CONDITION
C
 145  DO 451 I=1,2
      LI      = (I-1)*NAIRFO+1
      DGAMK(I,1) = AA(LI,1) + AA(LI+1,1)
      DGAMK(I,2) = AA(LI,2) + AA(LI+1,2)
  451 DGAMK(I,3) = -(BB(LI) + BB(LI+1))
C
C  SOLVE KUTTA CONDITION BY GAUSS
C
      R=DGAMK(2,1)/DGAMK(1,1)
      DO 461 K = 2,3
  461 DGAMK(2,K) = DGAMK(2,K)-R*DGAMK(1,K)
      GAMK(2) = DGAMK(2,3)/DGAMK(2,2)
      GAMK(1) = (DGAMK(1,3)-DGAMK(1,2)*GAMK(2))/DGAMK(1,1)
C
C     CALCULATE SOURCE STRENGTH
C
 300  DO 160  L = 1,NAIRFO
      LI      =(L-1)*NODTOT
      DO 160  I = 1,NODTOT
 160  QK(I+LI)   = GAMK(1)*B1(I+LI) + GAMK(2)*B2(I+LI) + B3(I+LI)
C
C     CALCULATE TANGENTIAL VELOCITY AT THE TRAILING EDGE BY BACK
C     SUBSTITUTION. THE PRODUCT OF THE TRAILING EDGE VELOCITY
C     SHOULD BE NEGATIVE . THIS IS CHECKED IN THE MAIN PROGRAM.
C
      VTAG1   = AA(1,1)*GAMK(1)+AA(1,2)*GAMK(2)+BB(1)
      VTAG50  = AA(2,1)*GAMK(1)+AA(2,2)*GAMK(2)+BB(2)
      VTAG51  = AA(3,1)*GAMK(1)+AA(3,2)*GAMK(2)+BB(3)
      VTA100 = AA(4,1)*GAMK(1)+AA(4,2)*GAMK(2)+BB(4)
      PVTAG   = VTAG1*VTAG50
 2222 FORMAT(1X,'VTAG',4F10.6)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C     SUBROUTINE TEWAK (SINALF,COSALF)                                C
C                                                                     C
C             COMPUTE WAKE ELEMENT AT THE TRAILING EDGE               C
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     VYWK(L) .... Y-VELO COMP AT THE MID-POINT OF THE WAKE ELEMENT
C                  WITH RESPECT TO THE LOCAL FROZEN FRAME OF REFERENCE
C     VXWK(L) .... X-VELO COMP AT THE MID-POINT OF THE WAKE ELEMENT
C                  WITH RESPECT TO THE LOCAL FROZEN FRAME OF REFERENCE
C     ____________________________________________________________
      SUBROUTINE TEWAK
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +             COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +             NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +             COSTHL(201),SINTHL(201)
      COMMON /COF/ A(201,211),KEQNS
      COMMON /SING/ Q(200),GAMMA(2),QK(200),GAMK(2)
      COMMON /WAK/ VYW(2),VXW(2),WAKE(2),DT
      COMMON /WAK2/ VYWK(2),VXWK(2)
      COMMON /CORV/ CV(2,200),XC(2,200),YC(2,200),M,TD,CVVX(2,200),
     +              CVVY(2,200),XCI(2,200),YCI(2,200)
      COMMON /INF1/ AAN(201,201),BBN(201,201),AYNP1(2,201),BYNP1(2,201),
     +              SUMAAN(201,2),SUMBBN(201,2)
      COMMON /INF2/ SUMCCN(201),SUMCCT(201),CYNP1(2,200),
     +              CXNP1(2,200)
      COMMON /GEOM/ SINALF(2),COSALF(2),OMEGA(2),UX(2),UY(2),PIVOT,
     +              XPRM,YPRM
      COMMON /GUST/ UG(200),VG(200),XGF,UGUST,VGUST
      DIMENSION CONT1X(2),CONT1Y(2),CONT2X(2),CONT2Y(2),
     + CONT3X(2),CONT3Y(2),CONT4X(2),CONT4Y(2),CONT5X(2),CONT5Y(2)
      DO 20 I = 1,NAIRFO
      NN      = (I-1)*NP1
      XMID    = 0.5 * (X(NP1+NN) + X(NP2+I-1))
      YMID    = 0.5 * (Y(NP1+NN) + Y(NP2+I-1))
      UGW     = 0.0
      VGW     = 0.0
      XG      = XMID
      IF (XG .GT. XGF) GO TO 10
      UGW     = UGUST
      VGW     = VGUST
 10   VYWK(I) = (1.+UGW)*SINALF(I)+VGW*COSALF(I)
      VXWK(I) = (1.+UGW)*COSALF(I)-VGW*SINALF(I)
 2221 FORMAT(1X,'SINALF :',4F10.5)
      CONT1Y(I) = (1.+UGW)*SINALF(I)+VGW*COSALF(I)
      CONT1X(I) = (1.+UGW)*COSALF(I)-VGW*SINALF(I)
      CONT2Y(I) = 0
      CONT2X(I) = 0
      CONT3Y(I) = 0
      CONT3X(I) = 0
C
C     CONTRIBUTION FROM THE SOURCE AND WAKE DIST FROM THE AIRFOIL
C
      DO 110 L = 1,NAIRFO
      KJ       = (L-1)*NODTOT
      DO 120  J = 1,NODTOT
      VYWK(I) = VYWK(I) + AYNP1(I,J+KJ)*QK(J+KJ) + BYNP1(I,J+KJ)*GAMK(L)
      VXWK(I) = VXWK(I) - BYNP1(I,J+KJ)*QK(J+KJ) + AYNP1(I,J+KJ)*GAMK(L)
      CONT2Y(I) = CONT2Y(I)+ AYNP1(I,J+KJ)*QK(J+KJ)
      CONT2X(I) = CONT2X(I)- BYNP1(I,J+KJ)*QK(J+KJ)
      CONT3Y(I) = CONT3Y(I)+ BYNP1(I,J+KJ)*GAMK(L)
  120 CONT3X(I) = CONT3X(I)+ AYNP1(I,J+KJ)*GAMK(L)
  110 CONTINUE
C
C     ADD WAKE ELEMENT CONTRIBUTION OF THE OTHER AIRFOIL
C
      J        = 3-I
      VYWK(I) = VYWK(I)+ BYNP1(I,NP3)*(GAMMA(J)-GAMK(J))
     + * SS(J)/WAKE(J)
      VXWK(I) = VXWK(I)+ AYNP1(I,NP3)*(GAMMA(J)-GAMK(J))
     + * SS(J)/WAKE(J)
      CONT4Y(I) =  BYNP1(I,NP3)*(GAMMA(J)-GAMK(J))
     + * SS(J)/WAKE(J)
  125 CONT4X(I) =  AYNP1(I,NP3)*(GAMMA(J)-GAMK(J))
     + * SS(J)/WAKE(J)
C
C        ADD CORE VORTEX CONTRIBUTION
C
      CONT5Y(I) =  0
      CONT5X(I) =  0
      IF (M .EQ. 1) GO TO 140
      MM1      = M - 1
      DO 150 L = 1,NAIRFO
      KN       =(L-1)*MM1
      DO 130  N = 1,MM1
      VYWK(I) = VYWK(I) + CYNP1(I,N+KN)*CV(L,N)
      VXWK(I) = VXWK(I) + CXNP1(I,N+KN)*CV(L,N)
      CONT5Y(I) =  CYNP1(I,N+KN)*CV(L,N)
  130 CONT5X(I) =  CXNP1(I,N+KN)*CV(L,N)
  150 CONTINUE
  140 CONTINUE
   20 CONTINUE
 2222 FORMAT (1X, 'VXWK :',6F10.5)
 2223 FORMAT (1X, 'VYWK :',6F10.5)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     SUBROUTINE PRESS (SINALF,COSALF,OMEGA,UX,UY)                     C
C                                                                      C
C             COMPUTE UNSTEADY FLOW PRESSURE DISTRIBUTION              C
C                  AND VELOCITY POTENTIAL AT MID-POINTS OF PANELS      C
C                                                                      C
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     VTANG  ..... TANGENTIAL VELO COMP OF A FIXED POINT OF THE MOVING
C                  LOCAL FRAME OF REFERENCE.
C     PHIK(I)..... DISTURBANCE VELO POTENTIAL AT THE MID POINT OF THE
C                  I-TH PANE AT THE CURRENT TIME STEP FOR THE UNSTEADY
C                  FLOW CASE.
C     PHI(I) ..... DISTURBANCE VELO POTENTIAL AT THE MID POINT OF THE
C                  I-TH PANEL FOR THE PREVIOUS TIME STEP
C     PHILE(L).... DIFFERENCE OF THE POTENTIALS OF THE LEADING EDGE TO
C                  THE LOWER TRAILING EDGE FOR THE RESPECTIVE AIRFOIL
C     PINK(L) .... DIFFERENCE OF THE POTENTIALS AT A POINT 100 CHORD
C                  LENGTH UPSTREAM OF THE LE FOR THE RESPECTIVE AIRFOIL
C     SUMC(L) .... GAMMA ASSOCIATED WITH THE INTEGRATION OF THE DISTURB-
C                  ANCE VELOCITY AROUND THE WHOLE AIRFOIL
C
C     THE FOLLOWING INFLUENCE COEF ARE COMPUTED FOR A FINER GRID ON THE
C     AIRFOIL SO AS TO OBTAIN A MORE ACCURATE VELO POTENTIAL AT THE TE
C     THE INFLUENCE COEF ON THE I-TH PANEL FROM THE J-TH PANEL OF THE
C     AIRFOIL REMAINS THE SAME FOR ALL TIME STEP
C                           ----
C
C   AANP1(I,J,K).. NORMAL VELOCITY INDUCED AT THE I-TH PANEL SUB NODE K
C                  OF AIRFOIL 1 DUE TO UNIT STRENGTH DIST SOURCE STRENG-
C                  TH ON THE J-TH PANEL OF AIRFOIL 1
C   BBNP1(I,J,K).. NORMAL VELOCITY INDUCED AT THE I-TH PANEL SUB NODE K
C                  OF AIRFOIL 1 DUE TO UNIT STRENGTH DIST VORTICITY STR-
C                  ENGTH ON THE J-TH PANEL OF AIRFOIL 1
C   AANP2(I,J,K).. NORMAL VELOCITY INDUCED AT THE I-TH PANEL SUB NODE K
C                  OF AIRFOIL 2 DUE TO UNIT STRENGTH DIST SOURCE STRENG-
C                  TH ON THE J-TH PANEL OF AIRFOIL 2
C   BBNP2(I,J,K).. NORMAL VELOCITY INDUCED AT THE I-TH PANEL SUB NODE K
C                  OF AIRFOIL 2 DUE TO UNIT STRENGTH DIST VORTICITY STR-
C                  ENGTH ON THE J-TH PANEL OF AIRFOIL 2
C
C
C
C
      SUBROUTINE PRESS
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +             COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +             NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +             COSTHL(201),SINTHL(201)
      COMMON /CPD/ CP(200),SCL,T,SCM,SGAM
      COMMON /NUM/ PI,PI2INV
      COMMON /SING/ Q(200),GAMMA(2),QK(200),GAMK(2)
      COMMON /WAK/ VYW(2),VXW(2),WAKE(2),DT
      COMMON /CORV/ CV(2,200),XC(2,200),YC(2,200),M,TD,CVVX(2,200),
     +              CVVY(2,200),XCI(2,200),YCI(2,200)
      COMMON /INF1/ AAN(201,201),BBN(201,201),AYNP1(2,201),BYNP1(2,201),
     +              SUMAAN(201,2),SUMBBN(201,2)
      COMMON /INF2/ SUMCCN(201),SUMCCT(201),CYNP1(2,200),
     +              CXNP1(2,200)
      COMMON /INFL3/ AANP(101,101,6),BBNP(101,101,6)
      COMMON /POT/ PHI(200),PHIK(200)
      COMMON /GUST/ UG(200),VG(200),XGF,UGUST,VGUST
      COMMON /EXTV/ UE(200),VN(200)
      COMMON /BMAT/ B1,B2,B3
      COMMON /GEOM/ SINALF(2),COSALF(2),OMEGA(2),UX(2),UY(2),PIVOT,
     +              XPRM,YPRM
      DIMENSION PHITEL(2),PHITEU(2),WGHT(5),PLOC(5),SUMC(2),
     +AANP1(50,50,9),AANP2(50,50,9),BBNP1(50,50,9),BBNP2(50,50,9),
     +COSTHP(102,6),SINTHP(102,6),AANP4(2),PHIL(102),UGU(100,6),
     +VGU(100,6),PDUM(2),DP(2),BBNP4(2),B1(200),B2(200),B3(200),PHILE(2)
 3333 DIMENSION CON2(2),VTGT(2,50),WAKCON(2,50),VOTCON(2,50),
     +        DUMCON(2,50)
 7733 DIMENSION CP1(200)
 4444 REAL *8 PINK(2),VELX
      DATA WGHT/.11846344,.23931434,.28444444,
     +          .23931434,.11846344/
      DATA PLOC/.04691008,.23076535,.50000000,
     +          .76923466,.95300899/
      WRITE (6,1000)
      IF (M .GT. 1)  GO TO 510
C
C     COMPUTE THE INFLUENCE COEFFICIENT FOR A FINER GRID
C     .... ON THE AIRFOIL BY THE SAME AIRFOIL ...
C               ( ONLY COMPUTED ONCE )
C
      DO 200  L = 1,NAIRFO
      LI      = (L-1)*NODTOT
      KI      = (L-1)*NP1
      DO 200  I = 1,NODTOT
      DO 200  K = 1,5
      DX      = PLOC(K)*(X(I+KI+1)-X(I+KI))
      DY      = PLOC(K)*(Y(I+KI+1)-Y(I+KI))
      XMID    = X(I+KI) + DX
      YMID    = Y(I+KI) + DY
      DO 200  J = 1,NODTOT
      FLOG    = 0.0
      FTAN    = PI
      IF (J+LI .EQ. I+LI)     GO TO 100
      DXJ     = XMID - X(J+KI)
      DXJP    = XMID - X(J+KI+1)
      DYJ     = YMID - Y(J+KI)
      DYJP    = YMID - Y(J+KI+1)
      FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
 100  CTIMTJ  = COSTHE(I+KI)*COSTHE(J+KI) +
     +          SINTHE(I+KI)*SINTHE(J+KI)
      STIMTJ  = SINTHE(I+KI)*COSTHE(J+KI) -
     +          COSTHE(I+KI)*SINTHE(J+KI)
      IF (L .EQ. 2) GOTO 120
      AANP1(I,J,K) = PI2INV*(FTAN*CTIMTJ + FLOG*STIMTJ)
      BBNP1(I,J,K) = PI2INV*(FLOG*CTIMTJ - FTAN*STIMTJ)
      GOTO 200
 120  AANP2(I,J,K) = PI2INV*(FTAN*CTIMTJ + FLOG*STIMTJ)
      BBNP2(I,J,K) = PI2INV*(FLOG*CTIMTJ - FTAN*STIMTJ)
 200  CONTINUE
 510  CONTINUE
C
```

```
C     FIND TANGENTIAL VELOCITY AT THE MIDPOINT OF THE I-TH PANEL
C
      DO 600 L = 1,NAIRFO
      LI=(L-1)*NODTOT
      KI=(L-1)*NP1
      SUMC(L)  =0.0
      DO 600  I = 1,NODTOT
      CONTR1      = 0.0
      CONTR2      = 0.0
      WAKCON(L,I) = 0.0
      VOTCON(L,I) = 0.0
      DUMCON(L,I) = 0.0
      XMID    = 0.5 * (XI(I+KI) + XI(I+KI+1))
      YMID    = 0.5 * (YI(I+KI) + YI(I+KI+1))
      DX      = (XI(I+KI+1) - XI(I+KI))
      DY      = (YI(I+KI+1) - YI(I+KI))
      DIST    = SQRT(DX*DX+DY*DY)
C
C     ACCOUNT FOR THE FREESTREAM AND THE MOTION OF THE AIRFOIL
C
      VSX     = (1.+UG(I+LI))*COSALF(L)-VG(I+LI)*SINALF(L) + OMEGA(L)*
     +          YMID + UX(L)
      VSY     = (1.+UG(I+LI))*SINALF(L)+VG(I+LI)*COSALF(L)
     + - OMEGA(L)*XMID + UY(L)
      VS      = VSX*VSX + VSY*VSY
      VNORM   = -VSX*SINTHL(I+KI)+VSY*COSTHL(I+KI)
      VTANG   = (((1.+UG(I+LI))*COSALF(L)-VG(I+LI)*SINALF(L)+UX(L))
     + *COSTHL(I+KI)+ ((1.+UG(I+LI))*SINALF(L)+VG(I+LI)*COSALF(L)+UY(L))
     + *SINTHL(I+KI)+ OMEGA(L)*(YMID*COSTHL(I+KI)
     +   - XMID*SINTHL(I+KI)))
      VTFREE  = VTANG
      VACT    = VTANG
C
C     INTRODUCE SMALLER GRIDS FOR THE PURPOSE OF THE VELO POTENTIAL.
C     VELO ONLY CALCULATED AT THE MIDPT OF THE PANEL WHERE K = 3
C
      DO 260  K = 1,5
      DX      =  PLOC(K)*(X(I+KI+1)-X(I+KI))
      DY      =  PLOC(K)*(Y(I+KI+1)-Y(I+KI))
      XINT    = X(I+KI) + DX
      YINT    = Y(I+KI) + DY
      VDUM    = 0.0
C
C     INFLUENCE COEF ON I-TH PANEL DUE TO THE WAKE ELEMENTS
C
      DO 245 MM = 1, NAIRFO
      J       = NP1*MM
      DXJ     = XINT - X(J)
      DXJP    = XINT - X(NAIRFO*NP1+MM)
      DYJ     = YINT - Y(J)
      DYJP    = YINT - Y(NAIRFO*NP1+MM)
      FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
      CTIMTJ  = COSTHE(I+KI)*COSTHE(J) +
     +          SINTHE(I+KI)*SINTHE(J)
      STIMTJ  = SINTHE(I+KI)*COSTHE(J) -
     +          COSTHE(I+KI)*SINTHE(J)
      AANP4(MM) = PI2INV*(FTAN*CTIMTJ + FLOG*STIMTJ)
  245 BBNP4(MM) = PI2INV*(FLOG*CTIMTJ - FTAN*STIMTJ)
      CONTR1 = SS(1)*(GAMMA(1)-GAMK(1))*AANP4(1)
     +/WAKE(1)+SS(2)*(GAMMA(2)-GAMK(2))*AANP4(2)/WAKE(2)
C
C     CONTRIBUTION TO VELO COMPONENT BY WAKE ELEMENT.
C
      IF (K .EQ. 3) THEN
      VACT = VACT + SS(1)*(GAMMA(1)-GAMK(1))*AAN(I+LI,
     +NP3)/WAKE(1)+SS(2)*(GAMMA(2)-GAMK(2))*AAN(I+LI,NP4)/WAKE(2)
      VNORM = VNORM + SS(1)*(GAMMA(1)-GAMK(1))*BBNP4(1)
     +/WAKE(1)+SS(2)*(GAMMA(2)-GAMK(2))*BBNP4(2)/WAKE(2)
      ENDIF
C
C               EFFECTS ON AIRFOIL BY THE SAME AIRFOIL
C              ----------------------------------------
C
C     INTEGRATION AROUND FIRST AIRFOIL
C     CONTRIBUTION TO VELO COMP BY AIRFOIL 1 WHEN K = 3
C
      DO 300 J = 1,NODTOT
      IF (L .EQ. 2) GOTO 270
      VDUM     = VDUM-BBNP1(I,J,K)*QK(J)+AANP1(I,J,K)*GAMK(1)
      IF (K .EQ. 3) THEN
      VACT = VACT-BBN(I+LI,J)*QK(J)+
     +AAN(I+LI,J)*GAMK(1)
      VNORM = VNORM+(AANP1(I,J,K)*QK(J))+
     +(BBNP1(I,J,K)*GAMK(1))
      ENDIF
      GOTO 300
C
C     INTEGRATION AROUND SECOND AIRFOIL
C     CONTRIBUTION TO VELO COMP BY AIRFOIL 2 WHEN K = 3
C
  270 VDUM     = VDUM-BBNP2(I,J,K)*QK(J+NODTOT)+AANP2(I,J,K)*GAMK(2)
      IF (K .EQ. 3) THEN
      VACT = VACT-BBN(I+LI,J+NODTOT)*QK(J+NODTOT)+
     + AAN(I+LI,J+NODTOT)*GAMK(2)
      VNORM = VNORM+(AANP2(I,J,K)*QK(J+NODTOT))+
     + (BBNP2(I,J,K)*GAMK(2))
      ENDIF
  300 CONTINUE
C
C               EFFECTS ON AIRFOIL BY THE OTHER AIRFOIL
C              ----------------------------------------
C
C     INTEGRATION AROUND BOTH AIRFOIL
C     CONTRIBUTION TO VELO COMP BY BOTH AIRFOIL WHEN K = 3
C
      DO 350  J = NODTOT+2,2*NODTOT+1
      DXJ     = XINT - X(J-KI)
      DXJP    = XINT - X(J-KI+1)
      DYJ     = YINT - Y(J-KI)
      DYJP    = YINT - Y(J-KI+1)
      FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
      CTIMTJ  = COSTHE(I+KI)*COSTHE(J-KI) +
     +          SINTHE(I+KI)*SINTHE(J-KI)
      STIMTJ  = SINTHE(I+KI)*COSTHE(J-KI) -
     +          COSTHE(I+KI)*SINTHE(J-KI)
      AANP3   = PI2INV*(FTAN*CTIMTJ + FLOG*STIMTJ)
      BBNP3   = PI2INV*(FLOG*CTIMTJ - FTAN*STIMTJ)
      IF (K.EQ.3) THEN
      VACT = VACT-BBN(I+LI,J-LI-1)*QK(J-LI-1)+
     +        AAN(I+LI,J-LI-1)*GAMK(3-L)
      VNORM = VNORM+(AANP3*QK(J-LI-1))+
     +        (BBNP3*GAMK(3-L))
      ENDIF
 350  VDUM    = VDUM - BBNP3*QK(J-LI-1) + AANP3*GAMK(3-L)
C
C     CONTRIBUTION BY CORE VORTICES
C     1. TO VELO POTENTIAL
C     2. TO VELO COMPONENT ONLY WHEN K = 3
C
      IF (M.EQ.1) GOTO 150
      MM1     = M - 1
      SUMCN   = 0.0
      SUMCT   = 0.0
      DO 400 MM = 1, NAIRFO
      KN      = (MM-1)*MM1
      DO 400  N = 1,MM1
      DXC     = XINT - XC(MM,N)
      DYC     = YINT - YC(MM,N)
      DISTC   = SQRT(DXC*DXC+DYC*DYC)
      COSTHN  = DXC/DISTC
      SINTHN  = DYC/DISTC
      CTIMTN  = COSTHE(I+KI)*COSTHN + SINTHE(I+KI)*SINTHN
      STIMTN  = SINTHE(I+KI)*COSTHN - COSTHE(I+KI)*SINTHN
      CCN     = -CTIMTN/DISTC
      CCT     = -STIMTN/DISTC
      SUMCN   = SUMCN + CCN*CV(MM,N)
 400  SUMCT   = SUMCT + CCT*CV(MM,N)
      IF (K .EQ. 3) THEN
      VACT = VACT+SUMCT*PI2INV
      VNORM = VNORM+(PI2INV*SUMCN)
      ENDIF
      CONTR2 = PI2INV*SUMCT
 150  CONTINUE
      VTANG   = VTANG + DBLE((CONTR1+CONTR2+VDUM)*WGHT(K))
      SUMC(L) = SUMC(L)+VDUM*DIST*WGHT(K)
      WAKCON(L,I) = WAKCON(L,I)+CONTR1
      VOTCON(L,I) = VOTCON(L,I)+CONTR2
      DUMCON(L,I) = DUMCON(L,I)+VDUM
 260  CONTINUE
      PHIK(I+LI) = ((VTANG)-VTFREE)*DIST
      CP(I+LI)   = VS - (VACT*VACT)
 7755 CP1(I+LI)  = CP(I+LI)
      UE(I+LI)   = VACT
      VN(I+LI)   = VNORM
 600  CONTINUE
 6688 FORMAT(2I5,7E15.7)
 3335 FORMAT (2I5,7F10.6)
C
C     COMPUTE DISTURBANCE POTENTIAL AT THE LEADING EDGE BY LINE
C              INTEGRAL OF THE VELOCITY FIELD
C        FROM UPSTREAM (AT INFINITY) TO THE LEADING EDGE
C
      TRY     = 1
      PINK(1) = 0.0
      PINK(2) = 0.0
 130  DO 56   L = 1,NAIRFO
      YMID    = PIVOT * SINALF(L) + (L-1)*YSHIFT
      XLE     = -PIVOT*COSALF(L)+(L-1)*XSHIFT
      XL      = XLE
C     XL      = 0.0
      NPHI    = 10 * NLOWER
      PDUM(L) = 0.0
      DO 30   I = 1,NPHI
      FRACT   = FLOAT(I)/FLOAT(NPHI)
      XLP     = -100.0 *TRY * (1.0 - COS(0.5*PI*FRACT))+XLE
      IF (I .EQ. 1) XLP = -0.000197+XLE
C     XLP     = -10.0 * (1.0 - COS(0.5*PI*FRACT))
      DELX    = XL - XLP
      XMID    = 0.5*(XL+XLP)
C     XMID    = 0.5*(XL+XLP)*COSALF(L)
C     YMID    = 0.5*(XL+XLP)*SINALF(L)
      XL      = XLP
      VELX    = UGUST
C
C     ADD CONTRIBUTION OF J-TH PANEL
C
      DO 40  MM = 1,NAIRFO
      LJ      =(MM-1)*NODTOT
      KJ      =(MM-1)*NP1
      DO 20   J = 1,NP1
      IF (J .EQ. NP1) GO TO 24
      DXJ     = XMID - X(J+KJ)
      DXJP    = XMID - X(J+KJ+1)
      DYJ     = YMID - Y(J+KJ)
      DYJP    = YMID - Y(J+KJ+1)
      FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
C     CALMTJ  = -COSTHE(J+KJ)
C     SALMTJ  = SINTHE(J+KJ)
      CALMTJ  = -COSALF(L)*COSTHE(J+KJ) - SINALF(L)*SINTHE(J+KJ)
      SALMTJ  = -SINALF(L)*COSTHE(J+KJ) + COSALF(L)*SINTHE(J+KJ)
      APY     = PI2INV*(FTAN*CALMTJ + FLOG*SALMTJ)
      BPY     = PI2INV*(FLOG*CALMTJ - FTAN*SALMTJ)
      VELX    = VELX  - DPROD(BPY,QK(J+LJ)) + DPROD(GAMK(MM),APY)
      GO TO 20
 24   DXJ     = XMID - X(J+KJ)
      DXJP    = XMID - X(2*NP1+MM)
      DYJ     = YMID - Y(J+KJ)
      DYJP    = YMID - Y(2*NP1+MM)
      FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
      CALMTJ  = -COSALF(L)*COSTHE(J+KJ) - SINALF(L)*SINTHE(J+KJ)
      SALMTJ  = -SINALF(L)*COSTHE(J+KJ) + COSALF(L)*SINTHE(J+KJ)
      APY     = PI2INV*(FTAN*CALMTJ + FLOG*SALMTJ)
      DUMMY   = SS(MM)*(GAMMA(MM)-GAMK(MM))/WAKE(MM)
      VELX    = VELX + DPROD(APY,DUMMY)
 20   CONTINUE
 40   CONTINUE
C
C     ADD CORE VORTEX CONTRIBUTION
C
      IF (M .EQ. 1) GO TO 50
      MM1     = M - 1
      DO 70  II = 1,NAIRFO
      KN      = (II-1)*MM1
      DO 60   N = 1,MM1
      DX      = XMID - XC(II,N)
      DY      = YMID - YC(II,N)
      DIST    = SQRT(DX*DX+DY*DY)
      COSTHN  = DX/DIST
      SINTHN  = DY/DIST
      SALMTN  = -SINALF(L)*COSTHN + COSALF(L)*SINTHN
      CPT     = -PI2INV*SALMTN/DIST
 60   VELX    = VELX  + DPROD(CPT,CV(II,N))
 70   CONTINUE
 50   CONTINUE
      PDUM(L) = PDUM(L) + VELX * DBLE(DELX)
 7771 FORMAT (2I5,6E14.6,2F10.6)
 30   CONTINUE
      DP(L) = PDUM(L)-PINK(L)
 56   CONTINUE
      PINK(1) = PDUM(1)
      PINK(2) = PDUM(2)
C
C     COMPUTATION OF THE VELOCITY POTENTIAL FOR MIDPOINT OF EACH PANEL
C
      DO 240 L   = 1,NAIRFO
      LI         = (L-1)*NODTOT
      PHP        = -PINK(L)
C
C     BEGIN WITH LOWER SURFACE
C
      DO 230 I   = NLOWER,1,-1
      PHC        = PHP-PHIK(I+LI)
      PHIK(I+LI) = 0.5*(PHP+PHC)
 230  PHP        = PHC
      PHITEL(L) = PHC
C
C     RESET FOR UPPER SURFACE
C
      PHP        = -PINK(L)
      DO 250 I   = NLOWER+1,NODTOT
      PHC        = PHP+PHIK(I+LI)
      PHIK(I+LI) = 0.5*(PHP+PHC)
 250  PHP        = PHC
      PHITEU(L) = PHC
 240  CONTINUE
```

```
C
C        COMPUTE CP AT MID POINT OF I-TH PANEL
C
      DO 295  L = 1,NAIRFO
      LI      = (L-1)*NODTOT
      KI      = (L-1)*NP1
      SUMC(L) = SUMC(L)/SS(L)
      DO 290  I = 1,NODTOT
      XIMID   = .5*(XI(I+KI) + XI(I+KI+1))
      XMID    = .5*(X(I+KI) + X(I+KI+1))
      YMID    = .5*(Y(I+KI) + Y(I+KI+1))
      CP(I+LI)= CP(I+LI) - 2.*(PHIK(I+LI)-PHI(I+LI))/DT
      WRITE (6,1050) I+LI,XIMID,XMID,YMID,QK(I+LI),GAMK(L),CP(I+LI),
     +  UE(I+LI),VN(I+LI),PHIK(I+LI),PHI(I+LI),SUMC(L)
 290  CONTINUE
      WRITE (6,235) PINK(L)
 295  CONTINUE
 235  FORMAT (1X,'PHI AT LEADING EDGE =',F10.6,/)
 1000 FORMAT(/,4X,'J',4X,'XI(J)',5X,'X(J)',6X,'Y(J)',6X,'Q(J)',6X,
     +'GAMMA',4X,'CP(J)',7X,'V(J)',6X,'VN(J)',5X,'PHIK',6X,
     +'PHI', 6X,'INTGAMK',/)
 1050 FORMAT(I5,12F10.5)
 1200 FORMAT(1X,'LENGTH OF LEADING EDGE INTEGRATION IN CHORD =',F10.6/)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     SUBROUTINE CORVOR (SINALF,COSALF)                                C
C                                                                      C
C             COMPUTE THE LOCAL VELOCITIES OF CORE VORTICES            C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CCVX(I,N) .... X-VELO OF THE I-TH AIRFOIL, N-TH CORE VORTEX WITH
C                    RESPECT TO THE CURRENT FROZEN FRAME OF REFERENCE
C     CCVY(I,N) .... Y-VELO OF THE I-TH AIRFOIL, N-TH CORE VORTEX WITH
C                    RESPECT TO THE CURRENT FROZEN FRAME OF REFERENCE
C     XC(I,N)   .... X-COORD OF THE LOCATION OF THE I-TH AIRFOIL, N-TH
C                    CORE VORTEX  W.R.T. THE GLOBAL FRAME OF REFERENCE.
C     YC(I,N)   .... Y-COORD OF THE LOCATION OF THE I-TH AIRFOIL, N-TH
C                    CORE VORTEX  W.R.T. THE GLOBAL FRAME OF REFERENCE.
C     GAMY(I)   .... GLOBAL Y-VELO AT A LOCATION OF THE I-TH AIRFOIL
C                    CORE VORTEX DUE TO A SOURCE DIST OF UNIT STRENGTH
C                    ON ONE PANEL
C     GBMY(I)   .... GLOBAL Y-VELO AT A LOCATION OF THE I-TH AIRFOIL
C                    CORE VORTEX DUE TO A VORTEX DIST OF UNIT STRENGTH
C                    ON ONE PANEL
C     AMY(I)    .... LOCAL Y-VELO AT A LOCATION OF THE I-TH AIRFOIL
C                    CORE VORTEX DUE TO A SOURCE DIST OF UNIT STRENGTH
C                    ON ONE PANEL
C     BMY(I)    .... LOCAL Y-VELO AT A LOCATION OF THE I-TH AIRFOIL
C                    CORE VORTEX DUE TO A VORTEX DIST OF UNIT STRENGTH
C                    ON ONE PANEL
C     SUMAMY(I) .... LOCAL Y-VELO AT A LOCATION OF THE I-TH AIRFOIL
C                    CORE VORTEX DUE TO A SOURCE DIST OF UNIT STRENGTH
C                    ON ALL PANELS
```

```
C     SUMBMY(I) .... LOCAL Y-VELO AT A LOCATION OF THE I-TH AIRFOIL
C                    CORE VORTEX DUE TO A VORTEX DIST OF UNIT STRENGTH
C                    ON ALL PANELS
C     GCMX      .... GLOBAL X-VELO AT A LOCATION OF A CORE VORTEX DUE
C                    TO  ANOTHER POINT VORTEX OF UNIT STRENGTH
C     GCMY      .... GLOBAL Y-VELO AT A LOCATION OF A CORE VORTEX DUE
C                    TO  ANOTHER POINT VORTEX OF UNIT STRENGTH
C     CMX       .... LOCAL X-VELO AT A LOCATION OF A CORE VORTEX DUE
C                    TO  ANOTHER POINT VORTEX OF UNIT STRENGTH
C     CMY       .... LOCAL Y-VELO AT A LOCATION OF A CORE VORTEX DUE
C                    TO  ANOTHER POINT VORTEX OF UNIT STRENGTH
C     ------------------------------------------------------------------
C
      SUBROUTINE CORVOR
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +             COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +             NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +             COSTHL(201),SINTHL(201)
      COMMON /SING/ Q(200),GAMMA(2),QK(200),GAMK(2)
      COMMON /WAK/ VYW(2),VXW(2),WAKE(2),DT
      COMMON /CORV/ CV(2,200),XC(2,200),YC(2,200),M,TD,CCVX(2,200),
     +              CCVY(2,200),XCI(2,200),YCI(2,200)
      COMMON /POT/ PHI(200),PHIK(200)
      COMMON /GUST/ UG(200),VG(200),XGF,UGUST,VGUST
      COMMON /NUM/ PI,PI2INV
      COMMON /GEOM/ SINALF(2),COSALF(2),OMEGA(2),UX(2),UY(2),PIVOT,
     +              XPRM,YPRM
      DIMENSION AMY(2),BMY(2),SUMAMY(2),SUMBMY(2),
     +VX(2),VY(2),GAMY(2),GBMY(2)
      IF (M.EQ.1) GOTO 40
      MM1     = M - 1
C
C     ACCOUNT FOR THE FREE STREAM INCLUDING GUST EFFECTS
C
      UGC     = 0.0
      VGC     = 0.0
      DO 15   I = 1,NAIRFO
      KN      = (I-1)*MM1
      DO 10   N = 1,MM1
      XG      = XC(I,N)*COSALF(I) + YC(I,N)*SINALF(I)
      IF (XG .GT. XGF) GO TO 5
      UGC     = UGUST
      VGC     = VGUST
 5    CONTINUE
      VY(I)   = (1.+UGC)*SINALF(I)+VGC*COSALF(I)
      VX(I)   = (1.+UGC)*COSALF(I)-VGC*SINALF(I)
      XMID    = XC(I,N)
      YMID    = YC(I,N)
C
C     CALCULATE THE INFLUENCE COEFFICIENT DUE TO THE AIRFOILS
C
      DO 25   L = 1,NAIRFO
      LJ      =(L-1)*NODTOT
      KJ      =(L-1)*NP1
      SUMAMY(I) = 0.0
      SUMBMY(I) = 0.0
      DO 20   J = 1,NP1
      DXJ     = XMID - X(J+KJ)
      DYJ     = YMID - Y(J+KJ)
      IF (J .EQ. NP1) GO TO 11
      DXJP    = XMID - X(J+KJ+1)
      DYJP    = YMID - Y(J+KJ+1)
      GO TO 12
 11   DXJP    = XMID - X(2*NP1+L)
      DYJP    = YMID - Y(2*NP1+L)
 12   FLOG    = .5*ALOG((DXJP*DXJP+DYJP*DYJP)/(DXJ*DXJ+DYJ*DYJ))
      FTAN    = ATAN2(DYJP*DXJ-DXJP*DYJ,DXJP*DXJ+DYJP*DYJ)
      GAMY(I) = PI2INV*(FTAN*COSTHE(J+KJ) - FLOG*SINTHE(J+KJ))
      GBMY(I) = PI2INV*(FLOG*COSTHE(J+KJ) + FTAN*SINTHE(J+KJ))
      AMY(I)  = GAMY(I)*COSALF(I)-GBMY(I)*SINALF(I)
      BMY(I)  = GAMY(I)*SINALF(I)+GBMY(I)*COSALF(I)
      IF (J.EQ.NP1) GOTO 20
      SUMAMY(I) = SUMAMY(I) + AMY(I)
      SUMBMY(I) = SUMBMY(I) + BMY(I)
      VY(I)      = VY(I) + AMY(I)*QK(J+LJ)
      VX(I)      = VX(I) - BMY(I)*QK(J+LJ)
 20   CONTINUE
C
C     ADD THE CONTRIBUTION DUE TO THE WAKE ELEMENTS
C
      VY(I)   = VY(I) + SUMBMY(I)*GAMK(L) + SS(L)*BMY(I)*
     +          (GAMMA(L)-GAMK(L))/WAKE(L)
      VX(I)   = VX(I) + SUMAMY(I)*GAMK(L) + SS(L)*AMY(I)*
     +          (GAMMA(L)-GAMK(L))/WAKE(L)
 25   CONTINUE
C
C     CALCULATE INFLUENCE COEFFICIENT DUE TO THE WAKE
C
      DO 35   L = 1,NAIRFO
      KMC     = (L-1)*MM1
      DO 30   MC = 1,MM1
      IF ((L.EQ.I) .AND. (MC.EQ.N)) GOTO 30
      DX      = XMID - XC(L,MC)
      DY      = YMID - YC(L,MC)
      DIST2   = DX*DX+DY*DY
      GCMY    = -PI2INV*DX/DIST2
      GCMX    = +PI2INV*DY/DIST2
      CMY     = GCMY*COSALF(I)+GCMX*SINALF(I)
      CMX     = -GCMY*SINALF(I)+GCMX*COSALF(I)
C
C     ADD CORE VORTEX CONTRIBUTION
C
      VY(I)   = VY(I) + CMY*CV(L,MC)
      VX(I)   = VX(I) + CMX*CV(L,MC)
 30   CONTINUE
 35   CONTINUE
C
C     CONVECTION VELOCITY OF CORE VORTICES AT NEXT TIME STEP
C
      CCVX(I,N) = VX(I)
      CCVY(I,N) = VY(I)
 10   CONTINUE
 15   CONTINUE
 40   CONTINUE
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     SUBROUTINE NEWPOSN                                               C
C                                                                      C
C     TRANSFORM THE RESPECTIVE FROZEN LOCAL COORDINATES AND PANEL      C
C     ANGLES TO THE GLOBAL FRAMES OF REFERENCE                         C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     XSHIFT .... GLOBAL X LOCATION OF THE 2-ND AIRFOIL PIVOT POSITION
C     YSHIFT .... GLOBAL Y LOCATION OF THE 2-ND AIRFOIL PIVOT POSITION
C     ITYPE  .... FLAG FOR DEALING WITH THE VARIOUS COMPONENTS
C                 = 0 : FOR TRANSFORMING AIRFOIL COORDINATES
C                 = 1 : FOR TRANSFORMING WAKE ELEMENTS
C                 = 2 : FOR TRANSFORMING MOST RECENT CORE VORTEX SHED
C                 = 3 : FOR TRANSFORMING ALL PREVIOUS CORE VORTICES
C     XPRM   .... X GLOBAL RELATIVE MOVEMENT OF THE PIVOTS POSITION
C     YPRM   .... Y GLOBAL RELATIVE MOVEMENT OF THE PIVOTS POSITION
C     _________________________________________________________________
C
      SUBROUTINE NEWPOS(ITYPE)
      COMMON /BOD/ IFLAG,NLOWER,NUPPER,NODTOT,X(202),Y(202),
     +             COSTHE(201),SINTHE(201),SS(2),NP1,NP2,NP3,NP4,
     +             NP5,XSHIFT,YSHIFT,NAIRFO,XI(202),YI(202),
     +             COSTHL(201),SINTHL(201)
      COMMON /CORV/ CV(2,200),XC(2,200),YC(2,200),M,TD,CVVX(2,200),
     +              CVVY(2,200),XCI(2,200),YCI(2,200)
      COMMON /GEOM/ SINALF(2),COSALF(2),OMEGA(2),UX(2),UY(2),PIVOT,
     +              XPRM,YPRM
C
C     CHECK FLAG TO ASCERTAIN TYPE OF TRANSFORMATION
C
      IF ( ITYPE .EQ. 1 ) GOTO 30
      IF ( ITYPE .EQ. 2 ) GOTO 40
      IF ( ITYPE .EQ. 3 ) GOTO 45
C
C     AIRFOIL COORDINATES AND LOCAL ANGLES TRANSFORMATION
C
      DO 10 I = 1,NP1
      X(I)    = (XI(I)-PIVOT)*COSALF(1)+YI(I)*SINALF(1)
      Y(I)    = (XI(I)-PIVOT)*(-SINALF(1))+YI(I)*COSALF(1)
   10 CONTINUE
      DO 20 I = NP1+1,2*NP1
      X(I)    = (XI(I)-PIVOT)*COSALF(2)+YI(I)*SINALF(2)+
     +          XSHIFT + XPRM
      Y(I)    = (XI(I)-PIVOT)*(-SINALF(2))+YI(I)*COSALF(2)+
     +          YSHIFT + YPRM
   20 CONTINUE
C
C     TRANSFORM LOCAL ANGLES
C
      DO 222 L  = 1,2
      K       = (L-1)*(NODTOT+1)
      DO 222  J = 1,NODTOT
      DX      = X(J+1+K) - X(J+K)
      DY      = Y(J+1+K) - Y(J+K)
      DIST    = SQRT(DX*DX +DY*DY)
      SINTHE(J+K) = DY/DIST
  222 COSTHE(J+K) = DX/DIST
      GOTO 50
C
C     WAKE ELEMENT TRANSFORMATION
C
   30 I       = NP2
      X(I)    = (XI(I)-PIVOT)*COSALF(1)+YI(I)*SINALF(1)
      Y(I)    = (XI(I)-PIVOT)*(-SINALF(1))+YI(I)*COSALF(1)
      I       = NP2 + 1
      X(I)    = (XI(I)-PIVOT)*COSALF(2)+YI(I)*SINALF(2)+
     +          XSHIFT + XPRM
      Y(I)    = (XI(I)-PIVOT)*(-SINALF(2))+YI(I)*COSALF(2)+
     +          YSHIFT + YPRM
C
C     WAKE ELEMENT ANGLES TRANSFORMATION
C
      DO 223 L  = 1,2
      K       = (L-1)*(NODTOT+1)
      J       = NP1
      DX      = X(NP2+L-1) - X(NP1*L)
      DY      = Y(NP2+L-1) - Y(NP1*L)
      DIST    = SQRT(DX*DX +DY*DY)
      SINTHE(J+K) = DY/DIST
  223 COSTHE(J+K) = DX/DIST
      GOTO 50
C
C     MOST RECENT CORE VORTEX SHED TRANSFORMATION
C
   40 XC(1,M) = (XCI(1,M)-PIVOT)*COSALF(1)+YCI(1,M)*SINALF(1)
      YC(1,M) = (XCI(1,M)-PIVOT)*(-SINALF(1))+YCI(1,M)*COSALF(1)
      XC(2,M) = (XCI(2,M)-PIVOT)*COSALF(2)+YCI(2,M)*SINALF(2)+
     +          XSHIFT + XPRM
      YC(2,M) = (XCI(2,M)-PIVOT)*(-SINALF(2))+YCI(2,M)*COSALF(2)+
     +          YSHIFT + YPRM
      GOTO 50
C
C     ALL PREVIOUS CORE VORTICES TRANFORMATION
C
   45 DO 46   I = 1,M-1
      XC(1,I) = (XCI(1,I)-PIVOT)*COSALF(1)+YCI(1,I)*SINALF(1)
      YC(1,I) = (XCI(1,I)-PIVOT)*(-SINALF(1))+YCI(1,I)*COSALF(1)
      XC(2,I) = (XCI(2,I)-PIVOT)*COSALF(2)+YCI(2,I)*SINALF(2)+
     +          XSHIFT + XPRM
      YC(2,I) = (XCI(2,I)-PIVOT)*(-SINALF(2))+YCI(2,I)*COSALF(2)+
     +          YSHIFT + YPRM
   46 CONTINUE
   50 CONTINUE
      RETURN
      END
```

APPENDIX B. EXAMPLE INPUT DATA FOR PROGRAM USPOTF2

```
   5
  ******************************************************************
        AIRFOIL TYPE : 8.4% THICKNESS SYMMETRICAL MISES AIRFOIL
        AIRFOIL COORDINATES ARE USER INPUT DATA (IFLAG = 1)
        NLOWER = 25 , NUPPER = 25
  ******************************************************************
  01   25   25
   2  0.0        2.0
 1.000000  0.994858  0.980866  0.958884  0.929536  0.893455
 0.851308  0.803815  0.751753  0.695948  0.637271  0.576620
 0.514918  0.453098  0.392084  0.332794  0.276105  0.222865
 0.173861  0.129819  0.091393  0.059146  0.033560  0.015010
 0.003767  0.000000  0.003767  0.015008  0.033560  0.059146
 0.091393  0.129819  0.173861  0.222865  0.276105  0.332791
 0.392082  0.453095  0.514915  0.576617  0.637266  0.695946
 0.751750  0.803815  0.851308  0.893455  0.929536  0.958884
 0.980866  0.994858  1.000000
 0.000000 -0.000782 -0.002784 -0.005721 -0.009351 -0.013459
-0.017837 -0.022285 -0.026618 -0.030671 -0.034289 -0.037341
-0.039712 -0.041314 -0.042083 -0.041979 -0.040979 -0.039096
-0.036360 -0.032820 -0.028555 -0.023651 -0.018220 -0.012379
-0.006259  0.000000  0.006259  0.012379  0.018220  0.023651
 0.028555  0.032820  0.036360  0.039096  0.040979  0.041979
 0.042083  0.041314  0.039712  0.037341  0.034289  0.030671
 0.026618  0.022285  0.017837  0.013459  0.009351  0.005721
 0.002784  0.000782  0.000000
 1.000000  0.994858  0.980866  0.958884  0.929536  0.893455
 0.851308  0.803815  0.751753  0.695948  0.637271  0.576620
 0.514918  0.453098  0.392084  0.332794  0.276105  0.222865
 0.173861  0.129819  0.091393  0.059146  0.033560  0.015010
 0.003767  0.000000  0.003767  0.015008  0.033560  0.059146
 0.091393  0.129819  0.173861  0.222865  0.276105  0.332791
 0.392082  0.453095  0.514915  0.576617  0.637266  0.695946
 0.751750  0.803815  0.851308  0.893455  0.929536  0.958884
 0.980866  0.994858  1.000000
 0.000000 -0.000782 -0.002784 -0.005721 -0.009351 -0.013459
-0.017837 -0.022285 -0.026618 -0.030671 -0.034289 -0.037341
-0.039712 -0.041314 -0.042083 -0.041979 -0.040979 -0.039096
-0.036360 -0.032820 -0.028555 -0.023651 -0.018220 -0.012379
-0.006259  0.000000  0.006259  0.012379  0.018220  0.023651
 0.028555  0.032820  0.036360  0.039096  0.040979  0.041979
 0.042083  0.041314  0.039712  0.037341  0.034289  0.030671
 0.026618  0.022285  0.017837  0.013459  0.009351  0.005721
 0.002784  0.000782  0.000000
  0.0       0.0     -45.80000  0.00    0.0        .0000      0
  0.0       0.00      0.0      0.00    0.0        .000       0.0     0.0
  5.0000    0.065     0.001    0.00    4.855698  -.921370  1.216290    0
```

APPENDIX C. EXAMPLE OUTPUT DATA FROM PROGRAM USPOTF2

```
DATA READ FROM FILE CODE 1

   5
   *****************************************************************
         AIRFOIL TYPE : 8.4% THICKNESS SYMMETRICAL MISES AIRFOIL
         AIRFOIL COORDINATES ARE USER INPUT DATA (IFLAG = 1)
         NLOWER = 25 , NUPPER = 25
   *****************************************************************
   1   25   25
   2  0.0  2.0
 1.000000  0.994858  0.980866  0.958884  0.929536  0.893455
 0.851308  0.803815  0.751753  0.695948  0.637271  0.576620
 0.514918  0.453098  0.392084  0.332794  0.276105  0.222865
 0.173861  0.129819  0.091393  0.059146  0.033560  0.015010
 0.003767  0.000000  0.003767  0.015008  0.033560  0.059146
 0.091393  0.129819  0.173861  0.222865  0.276105  0.332791
 0.392082  0.453095  0.514915  0.576617  0.637266  0.695946
 0.751750  0.803815  0.851308  0.893455  0.929536  0.958884
 0.980866  0.994858  1.000000
 0.000000 -0.000782 -0.002784 -0.005721 -0.009351 -0.013459
-0.017837 -0.022285 -0.026618 -0.030671 -0.034289 -0.037341
-0.039712 -0.041314 -0.042083 -0.041979 -0.040979 -0.039096
-0.036360 -0.032820 -0.028555 -0.023651 -0.018220 -0.012379
-0.006259  0.000000  0.006259  0.012379  0.018220  0.023651
 0.028555  0.032820  0.036360  0.039096  0.040979  0.041979
 0.042083  0.041314  0.039712  0.037341  0.034289  0.030671
 0.026618  0.022285  0.017837  0.013459  0.009351  0.005721
 0.002784  0.000782  0.000000
 1.000000  0.994858  0.980866  0.958884  0.929536  0.893455
 0.851308  0.803815  0.751753  0.695948  0.637271  0.576620
 0.514918  0.453098  0.392084  0.332794  0.276105  0.222865
 0.173861  0.129819  0.091393  0.059146  0.033560  0.015010
 0.003767  0.000000  0.003767  0.015008  0.033560  0.059146
 0.091393  0.129819  0.173861  0.222865  0.276105  0.332791
 0.392082  0.453095  0.514915  0.576617  0.637266  0.695946
 0.751750  0.803815  0.851308  0.893455  0.929536  0.958884
 0.980866  0.994858  1.000000
 0.000000 -0.000782 -0.002784 -0.005721 -0.009351 -0.013459
-0.017837 -0.022285 -0.026618 -0.030671 -0.034289 -0.037341
-0.039712 -0.041314 -0.042083 -0.041979 -0.040979 -0.039096
-0.036360 -0.032820 -0.028555 -0.023651 -0.018220 -0.012379
-0.006259  0.000000  0.006259  0.012379  0.018220  0.023651
 0.028555  0.032820  0.036360  0.039096  0.040979  0.041979
 0.042083  0.041314  0.039712  0.037341  0.034289  0.030671
 0.026618  0.022285  0.017837  0.013459  0.009351  0.005721
 0.002784  0.000782  0.000000
 0.000000  0.000000-45.800003  0.000000  0.000000  0.000000  0.000000  0
 0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
 0.650000  0.025000  0.001000  0.000000  4.855698 -0.921370  1.216290  0
```

```
TOTAL NO OF AIRFOILS =    2


 AIRFOIL( 1) PERIMETER LENGTH =   2.018599


 AIRFOIL( 2) PERIMETER LENGTH =   2.018599


 XSHIFT =   0.0   YSHIFT =   2.0


 STEADY FLOW SOLUTION AT ALPHA( 1) =   0.000000


 STEADY FLOW SOLUTION AT ALPHA( 2) =   0.000000


 AIRFOIL NUMBER   1


    J    X(J)      Y(J)       Q(J)       GAMMA     CP(J)       V(J)       VN(J)       PHI     INTGAMMA

    1  0.997429 -0.000391 -0.126260  0.000930  0.311915 -0.829509 -0.000001  0.025931  0.000928
    2  0.987862 -0.001783 -0.129608  0.000930  0.194662 -0.897406 -0.000001  0.027049  0.000928
    3  0.969875 -0.004252 -0.127489  0.000930  0.115548 -0.940453 -0.000001  0.028291  0.000928
    4  0.944210 -0.007536 -0.122211  0.000930  0.057081 -0.971040  0.000000  0.029175  0.000928
    5  0.911495 -0.011405 -0.115628  0.000930  0.010778 -0.994596  0.000001  0.029464  0.000928
    6  0.872381 -0.015648 -0.107951  0.000930 -0.027550 -1.013681  0.000001  0.029020  0.000928
    7  0.827561 -0.020061 -0.099320  0.000930 -0.060235 -1.029677  0.000001  0.027769  0.000928
    8  0.777784 -0.024451 -0.089949  0.000930 -0.088420 -1.043274  0.000001  0.025692  0.000928
    9  0.723850 -0.028644 -0.080054  0.000930 -0.113102 -1.055037  0.000001  0.022803  0.000928
   10  0.666609 -0.032480 -0.069387  0.000930 -0.134909 -1.065321  0.000000  0.019151  0.000928
   11  0.606945 -0.035815 -0.058064  0.000930 -0.154208 -1.074341  0.000000  0.014808  0.000928
   12  0.545769 -0.038526 -0.045835  0.000930 -0.171274 -1.082254  0.000000  0.009870  0.000928
 13  0.484008 -0.040513 -0.032712  0.000930 -0.186271 -1.089161  0.000000  0.004454  0.000928
 14  0.422591 -0.041698 -0.018509  0.000930 -0.199385 -1.095164  0.000000 -0.001312  0.000928
 15  0.362439 -0.042031 -0.002972  0.000930 -0.210762 -1.100347  0.000000 -0.007294  0.000928
 16  0.304449 -0.041479  0.014557  0.000930 -0.220282 -1.104664  0.000000 -0.013348  0.000928
 17  0.249485 -0.040037  0.034208  0.000930 -0.227800 -1.108062  0.000000 -0.019331  0.000928
 18  0.198363 -0.037728  0.057056  0.000930 -0.233195 -1.110494  0.000000 -0.025099  0.000928
 19  0.151840 -0.034590  0.084616  0.000930 -0.235732 -1.111635  0.000001 -0.030521  0.000928
 20  0.110606 -0.030687  0.118752  0.000930 -0.234295 -1.110989  0.000000 -0.035469  0.000928
 21  0.075269 -0.026103  0.164270  0.000930 -0.226218 -1.107347 -0.000001 -0.039835  0.000928
 22  0.046353 -0.020935  0.229441  0.000930 -0.204746 -1.097610 -0.000001 -0.043526  0.000928
 23  0.024285 -0.015299  0.336260  0.000930 -0.149130 -1.071975 -0.000001 -0.046472  0.000928
 24  0.009388 -0.009319  0.552083  0.000930  0.026593 -0.986614 -0.000001 -0.048594  0.000928
 25  0.001884 -0.003130  1.101697  0.000930  0.686931 -0.559526  0.000000 -0.049610  0.000928
 26  0.001884  0.003130  1.104512  0.000930  0.689549  0.557181 -0.000002 -0.049619  0.000928
 27  0.009387  0.009319  0.556927  0.000930  0.028260  0.985769 -0.000002 -0.048617  0.000928
 28  0.024284  0.015299  0.341580  0.000930 -0.149136  1.071978 -0.000002 -0.046501  0.000928
 29  0.046353  0.020935  0.235201  0.000930 -0.205956  1.098161 -0.000002 -0.043548  0.000928
 30  0.075269  0.026103  0.170178  0.000930 -0.228379  1.108323 -0.000001 -0.039834  0.000928
 31  0.110606  0.030687  0.124606  0.000930 -0.237267  1.112326  0.000000 -0.035425  0.000928
 32  0.151840  0.034590  0.090221  0.000930 -0.239397  1.113282  0.000001 -0.030413  0.000928
 33  0.198363  0.037728  0.062221  0.000930 -0.237453  1.112409  0.000000 -0.024905  0.000928
 34  0.249485  0.040037  0.038754  0.000930 -0.232571  1.110212  0.000000 -0.019029  0.000928
 35  0.304448  0.041479  0.018319  0.000930 -0.225413  1.106984  0.000000 -0.012919  0.000928
 36  0.362436  0.042031 -0.000146  0.000930 -0.216156  1.102795  0.000000 -0.006721  0.000928
 37  0.422588  0.041698 -0.016743  0.000930 -0.204859  1.097661  0.000000 -0.000584  0.000928
 38  0.484005  0.040513 -0.032101  0.000930 -0.191730  1.091664  0.000000  0.005344  0.000928
 39  0.545766  0.038526 -0.046448  0.000930 -0.176608  1.084716  0.000000  0.010922  0.000928
 40  0.606941  0.035815 -0.059942  0.000930 -0.159279  1.076698  0.000000  0.016015  0.000928
 41  0.666606  0.032480 -0.072523  0.000930 -0.139642  1.067540  0.000000  0.020503  0.000928
 42  0.723848  0.028644 -0.084451  0.000930 -0.117396  1.057070  0.000001  0.024285  0.000928
 43  0.777782  0.024451 -0.095542  0.000930 -0.092220  1.045094  0.000001  0.027285  0.000928
 44  0.827561  0.020061 -0.106076  0.000930 -0.063466  1.031245  0.000001  0.029454  0.000928
 45  0.872381  0.015648 -0.115792  0.000930 -0.030221  1.014998  0.000001  0.030775  0.000928
 46  0.911495  0.011405 -0.124478  0.000930  0.008687  0.995647  0.000001  0.031270  0.000928
 47  0.944210  0.007536 -0.131992  0.000930  0.055535  0.971836 -0.000001  0.031016  0.000928
 48  0.969875  0.004252 -0.138138  0.000930  0.114552  0.940983 -0.000001  0.030151  0.000928
 49  0.987862  0.001783 -0.141108  0.000930  0.194186  0.897672 -0.000001  0.028918  0.000928
 50  0.997429  0.000391 -0.138690  0.000930  0.311916  0.829508 -0.000001  0.027803  0.000928
PHI AT LEADING EDGE =  0.049783
```

```
AIRFOIL NUMBER   2

  J    X(J)      Y(J)      Q(J)     GAMMA      CP(J)      V(J)      VN(J)      PHI     INTGAMMA

 51  0.997429  1.999609 -0.138694 -0.000930  0.311927 -0.829502 -0.000006  0.027804 -0.000925
 52  0.987862  1.998217 -0.141109 -0.000930  0.194176 -0.897677 -0.000008  0.028919 -0.000925
 53  0.969875  1.995747 -0.138138 -0.000930  0.114554 -0.940982 -0.000003  0.030152 -0.000925
 54  0.944210  1.992463 -0.131991 -0.000930  0.055532 -0.971837 -0.000003  0.031016 -0.000925
 55  0.911495  1.988594 -0.124477 -0.000930  0.008663 -0.995659 -0.000002  0.031271 -0.000925
 56  0.872381  1.984351 -0.115792 -0.000930 -0.030320 -1.015047 -0.000002  0.030775 -0.000925
 57  0.827561  1.979939 -0.106076 -0.000930 -0.063560 -1.031291 -0.000001  0.029454 -0.000925
 58  0.777784  1.975548 -0.095552 -0.000930 -0.092303 -1.045134 -0.000003  0.027285 -0.000925
 59  0.723850  1.971355 -0.084444 -0.000930 -0.117485 -1.057112 -0.000001  0.024284 -0.000925
 60  0.666609  1.967520 -0.072526 -0.000930 -0.139696 -1.067566 -0.000001  0.020502 -0.000925
 61  0.606945  1.964185 -0.059934 -0.000930 -0.159324 -1.076719 -0.000001  0.016013 -0.000925
 62  0.545769  1.961473 -0.046443 -0.000930 -0.176637 -1.084729 -0.000001  0.010919 -0.000925
 63  0.484008  1.959487 -0.032095 -0.000930 -0.191745 -1.091671  0.000000  0.005341 -0.000925
 64  0.422591  1.958301 -0.016737 -0.000930 -0.204864 -1.097663  0.000001 -0.000587 -0.000925
 65  0.362439  1.957968 -0.000141 -0.000930 -0.216151 -1.102793  0.000000 -0.006724 -0.000925
 66  0.304449  1.958520  0.018322 -0.000930 -0.225426 -1.106990  0.000000 -0.012923 -0.000925
 67  0.249485  1.959962  0.038759 -0.000930 -0.232575 -1.110214  0.000001 -0.019032 -0.000925
 68  0.198363  1.962271  0.062226 -0.000930 -0.237525 -1.112441  0.000001 -0.024909 -0.000925
 69  0.151840  1.965409  0.090224 -0.000930 -0.239484 -1.113321  0.000003 -0.030416 -0.000925
 70  0.110606  1.969312  0.124609 -0.000930 -0.237364 -1.112369  0.000003 -0.035429 -0.000925
 71  0.075269  1.973896  0.170186 -0.000930 -0.228475 -1.108366  0.000001 -0.039837 -0.000925
 72  0.046353  1.979064  0.235216 -0.000930 -0.206053 -1.098205  0.000004 -0.043551 -0.000925
 73  0.024285  1.984700  0.341674 -0.000930 -0.149199 -1.072007  0.000008 -0.046503 -0.000925
 74  0.009388  1.990681  0.556827 -0.000930  0.028241 -0.985778  0.000002 -0.048618 -0.000925
 75  0.001884  1.996870  1.104514 -0.000930  0.689556 -0.557175  0.000008 -0.049619 -0.000925
 76  0.001884  2.003129  1.101710 -0.000930  0.686925  0.559531  0.000006 -0.049610 -0.000925
 77  0.009387  2.009318  0.552192 -0.000930  0.026589  0.986616 -0.000001 -0.048593 -0.000925
 78  0.024284  2.015299  0.336170 -0.000930 -0.149252  1.072032 -0.000002 -0.046470 -0.000925
 79  0.046353  2.020935  0.229431 -0.000930 -0.204849  1.097656  0.000005 -0.043524 -0.000925
 80  0.075269  2.026102  0.164263 -0.000930 -0.226311  1.107389 -0.000002 -0.039832 -0.000925
 81  0.110606  2.030687  0.118745 -0.000930 -0.234395  1.111033  0.000004 -0.035466 -0.000925
 82  0.151840  2.034590  0.084610 -0.000930 -0.235830  1.111679  0.000002 -0.030517 -0.000925
```

```
  83  0.198363  2.037727  0.057052 -0.000930 -0.233242  1.110515  0.000000 -0.025095 -0.000925
  84  0.249485  2.040037  0.034204 -0.000930 -0.227802  1.108063  0.000001 -0.019326 -0.000925
  85  0.304448  2.041478  0.014555 -0.000930 -0.220288  1.104667 -0.000001 -0.013344 -0.000925
  86  0.362436  2.042030 -0.002977 -0.000930 -0.210762  1.100347  0.000000 -0.007289 -0.000925
  87  0.422588  2.041698 -0.018516 -0.000930 -0.199389  1.095166  0.000001 -0.001307 -0.000925
  88  0.484005  2.040512 -0.032718 -0.000930 -0.186277  1.089164 -0.000001  0.004459 -0.000925
  89  0.545766  2.038526 -0.045841 -0.000930 -0.171311  1.082272  0.000001  0.009876 -0.000925
  90  0.606941  2.035814 -0.058073 -0.000930 -0.154244  1.074358 -0.000001  0.014813 -0.000925
  91  0.666606  2.032479 -0.069385 -0.000930 -0.134953  1.065342 -0.000001  0.019156 -0.000925
  92  0.723848  2.028644 -0.080059 -0.000930 -0.113194  1.055080 -0.000001  0.022808 -0.000925
  93  0.777782  2.024450 -0.089938 -0.000930 -0.088508  1.043316 -0.000002  0.025697 -0.000925
  94  0.827561  2.020061 -0.099318 -0.000930 -0.060315  1.029716 -0.000002  0.027774 -0.000925
  95  0.872381  2.015647 -0.107950 -0.000930 -0.027614  1.013713  0.000000  0.029024 -0.000925
  96  0.911495  2.011404 -0.115627 -0.000930  0.010801  0.994585  0.000000  0.029468 -0.000925
  97  0.944210  2.007535 -0.122210 -0.000930  0.057082  0.971040 -0.000001  0.029179 -0.000925
  98  0.969875  2.004251 -0.127488 -0.000930  0.115552  0.940451  0.000000  0.028295 -0.000925
  99  0.987862  2.001782 -0.129606 -0.000930  0.194652  0.897412 -0.000008  0.027053 -0.000925
 100  0.997429  2.000390 -0.126253 -0.000930  0.311935  0.829497  0.000019  0.025935 -0.000925
PHI AT LEADING EDGE =  0.049783



AIRFOIL NO    1
   CD =  0.000192    CL=  0.003860    CM= -0.001899


AIRFOIL NO    2
   CD =  0.000190    CL= -0.003864    CM=  0.001904




***************************************
***  BEGIN UNSTEADY FLOW SOLUTION  ****
***************************************
```

```
TIME STEP TK =   0.025000          TK - TKM1 =   0.025000


ALPHA(1)  = -45.800003      ALPHA(2)  =  45.800003
OMEGA(1)  =   0.000000      OMEGA(2)  =   0.000000
U(1)      =   0.000000       U(2)     =   0.000000
V(1)      =   0.000000       V(2)     =   0.000000


 NITR   VXW(1)    VYW(1)   WAKE(1)  THETA(1)     GAMK(1)       VXW(2)    VYW(2)   WAKE(2)  THETA(2)   GAMK(2)

   0  1.000000  0.000000  0.025000  0.000000  0.929847E-03  1.000000  0.000000  0.025000  0.000000 -0.929827E-03
   0  1.000000  0.000000  0.025000  0.000000 -0.747133E-01  1.000000  0.000000  0.025000  0.000000  0.746955E-01
   1  0.686703 -2.606758  0.067392 -1.313215 -0.747133E-01  0.686580  2.606971  0.067397  1.313280  0.746955E-01
   2  0.232384 -1.304157  0.033117 -1.394460 -0.172768E+00  0.232264  1.304035  0.033114  1.394532  0.172775E+00
   3  0.098293 -1.672256  0.041879 -1.512084 -0.137173E+00  0.098118  1.672258  0.041878  1.512189  0.137171E+00
   4  0.137433 -1.471437  0.036946 -1.477666 -0.155151E+00  0.137279  1.471267  0.036941  1.477758  0.155156E+00
   5  0.113802 -1.564139  0.039207 -1.498166 -0.146307E+00  0.113634  1.564155  0.039207  1.498274  0.146304E+00
   6  0.124862 -1.517797  0.038073 -1.488715 -0.150606E+00  0.124712  1.517638  0.038069  1.488805  0.150614E+00
   7  0.119332 -1.540308  0.038623 -1.493477 -0.148490E+00  0.119162  1.540221  0.038621  1.493583  0.148490E+00
   8  0.122018 -1.529213  0.038352 -1.491173 -0.149525E+00  0.121857  1.529071  0.038348  1.491270  0.149529E+00
   9  0.120693 -1.534634  0.038484 -1.492311 -0.149018E+00  0.120529  1.534553  0.038482  1.492414  0.149018E+00
  10  0.121351 -1.531978  0.038419 -1.491749 -0.149267E+00  0.121185  1.531851  0.038416  1.491850  0.149270E+00
  11  0.121025 -1.533291  0.038451 -1.492027 -0.149144E+00  0.120858  1.533178  0.038448  1.492129  0.149146E+00
  12  0.121187 -1.532643  0.038436 -1.491889 -0.149205E+00  0.121019  1.532527  0.038432  1.491992  0.149206E+00


CONVERGED SOLUTION OBTAINED AFTER NITR =  12

  J    XI(J)      X(J)      Y(J)      Q(J)      GAMMA     CP(J)      V(J)      VN(J)     PHIK      PHI      INTGAMK

  1   0.99743   0.69565   0.71479 -12.24440  -0.14918 -44.26076  -2.33434   0.00033   0.52358   0.02593  -0.14776
  2   0.98786   0.68998   0.70697 -10.27545  -0.14918 -38.82001  -0.87099   0.00038   0.51532   0.02705  -0.14776
  3   0.96987   0.67921   0.69235 -10.70724  -0.14918 -38.27182  -0.00478   0.00025   0.51919   0.02829  -0.14776
  4   0.94421   0.66367   0.67166 -11.09609  -0.14918 -39.81499   0.25782  -0.00002   0.53853   0.02918  -0.14776
  5   0.91150   0.64364   0.64551 -11.11243  -0.14918 -41.99179   0.22406   0.00000   0.56623   0.02946  -0.14776
  6   0.87238   0.61941   0.61451 -10.89968  -0.14918 -44.34621   0.09687  -0.00002   0.59573   0.02902  -0.14776
  7   0.82756   0.59133   0.57930 -10.59397  -0.14918 -46.64963  -0.04780  -0.00001   0.62336   0.02777  -0.14776
```

8	0.77778	0.55977	0.54055	-10.26679	-0.14918	-48.75861	-0.18735	0.00006	0.64724	0.02569	-0.14776
9	0.72385	0.52518	0.49897	-9.95016	-0.14918	-50.58185	-0.31683	0.00006	0.66632	0.02280	-0.14776
10	0.66661	0.48802	0.45526	-9.65770	-0.14918	-52.06068	-0.43770	0.00005	0.68001	0.01915	-0.14776
11	0.60695	0.44882	0.41016	-9.39481	-0.14918	-53.15674	-0.55167	0.00005	0.68796	0.01481	-0.14776
12	0.54577	0.40811	0.36441	-9.16259	-0.14918	-53.84744	-0.66171	0.00005	0.68999	0.00987	-0.14776
13	0.48401	0.36648	0.31875	-8.96051	-0.14918	-54.12111	-0.77009	0.00005	0.68606	0.00445	-0.14776
14	0.42259	0.32451	0.27389	-8.78668	-0.14918	-53.97699	-0.87952	0.00005	0.67623	-0.00131	-0.14776
15	0.36244	0.28281	0.23053	-8.63898	-0.14918	-53.42468	-0.99304	0.00004	0.66069	-0.00729	-0.14776
16	0.30445	0.24199	0.18935	-8.51576	-0.14918	-52.48463	-1.11475	0.00005	0.63968	-0.01335	-0.14776
17	0.24948	0.20264	0.15095	-8.41562	-0.14918	-51.18811	-1.24866	0.00006	0.61353	-0.01933	-0.14776
18	0.19836	0.16534	0.11591	-8.33590	-0.14918	-49.58678	-1.40203	0.00004	0.58266	-0.02510	-0.14776
19	0.15184	0.13066	0.08474	-8.27463	-0.14918	-47.75774	-1.58543	0.00005	0.54753	-0.03052	-0.14776
20	0.11061	0.09911	0.05790	-8.22888	-0.14918	-45.82455	-1.81525	0.00005	0.50865	-0.03547	-0.14776
21	0.07527	0.07119	0.03576	-8.19127	-0.14918	-44.02069	-2.12355	0.00004	0.46656	-0.03984	-0.14776
22	0.04635	0.04732	0.01864	-8.14575	-0.14918	-42.83888	-2.57173	0.00005	0.42179	-0.04353	-0.14776
23	0.02428	0.02790	0.00674	-8.03369	-0.14918	-43.62502	-3.30520	0.00005	0.37479	-0.04647	-0.14776
24	0.00939	0.01323	0.00023	-7.57487	-0.14918	-51.14941	-4.71313	0.00007	0.32560	-0.04859	-0.14776
25	0.00188	0.00356	-0.00083	-4.45520	-0.14918	-80.27815	-7.43541	0.00010	0.27530	-0.04961	-0.14776
26	0.00188	-0.00093	0.00353	5.95508	-0.14918	-65.55803	-6.67630	0.00001	0.22519	-0.04962	-0.14776
27	0.00939	-0.00014	0.01323	8.32812	-0.14918	-28.34676	-3.37298	0.00001	0.17601	-0.04862	-0.14776
28	0.02428	0.00596	0.02808	8.49468	-0.14918	-16.46977	-1.84788	0.00000	0.12919	-0.04650	-0.14776
29	0.04635	0.01731	0.04783	8.46184	-0.14918	-10.48329	-1.07912	0.00001	0.08544	-0.04355	-0.14776
30	0.07527	0.03376	0.07216	8.41897	-0.14918	-6.14475	-0.61715	-0.00001	0.04471	-0.03983	-0.14776
31	0.11061	0.05511	0.10069	8.39495	-0.14918	-2.51735	-0.30317	0.00001	0.00739	-0.03543	-0.14776
32	0.15184	0.08106	0.13297	8.39446	-0.14918	0.64860	-0.07144	0.00001	-0.02608	-0.03041	-0.14776
33	0.19836	0.11124	0.16851	8.41841	-0.14918	3.41737	0.11190	0.00001	-0.05528	-0.02491	-0.14776
34	0.24948	0.14523	0.20677	8.46723	-0.14918	5.79068	0.26410	0.00001	-0.07978	-0.01903	-0.14776
35	0.30445	0.18251	0.24718	8.54081	-0.14918	7.74898	0.39641	0.00002	-0.09925	-0.01292	-0.14776
36	0.36244	0.22255	0.28914	8.64033	-0.14918	9.26321	0.51647	0.00001	-0.11335	-0.00672	-0.14776
37	0.42259	0.26472	0.33203	8.76705	-0.14918	10.30394	0.62870	0.00002	-0.12182	-0.00058	-0.14776
38	0.48400	0.30839	0.37523	8.92163	-0.14918	10.84198	0.73769	0.00002	-0.12448	0.00534	-0.14776
39	0.54577	0.35287	0.41812	9.10576	-0.14918	10.85084	0.84683	0.00002	-0.12118	0.01092	-0.14776
40	0.60694	0.39746	0.46009	9.32101	-0.14918	10.30816	0.95933	0.00002	-0.11184	0.01601	-0.14776
41	0.66661	0.44145	0.50054	9.56783	-0.14918	9.19853	1.07796	0.00003	-0.09650	0.02050	-0.14776
42	0.72385	0.48411	0.53890	9.84473	-0.14918	7.51157	1.20661	0.00003	-0.07531	0.02429	-0.14776
43	0.77778	0.52471	0.57465	10.14673	-0.14918	5.24719	1.34867	0.00002	-0.04854	0.02729	-0.14776
44	0.82756	0.56256	0.60727	10.46027	-0.14918	2.40967	1.50981	0.00001	-0.01666	0.02945	-0.14776
45	0.87238	0.59698	0.63633	10.75573	-0.14918	-0.98944	1.69642	0.00006	0.01967	0.03077	-0.14776
46	0.91150	0.62729	0.66141	10.96559	-0.14918	-4.94559	1.91964	0.00008	0.05953	0.03127	-0.14776
47	0.94421	0.65287	0.68217	10.95881	-0.14918	-9.52099	2.20630	-0.00006	0.10168	0.03102	-0.14776

```
  48   0.96987   0.67311   0.69828  10.56902  -0.14918 -15.13123   2.64243   0.00009   0.14451   0.03015  -0.14776
  49   0.98786   0.68742   0.70945   9.96707  -0.14918 -23.68739   3.48605   0.00030   0.18560   0.02892  -0.14776
  50   0.99743   0.69509   0.71534  11.37057  -0.14918 -43.91580   5.44891   0.00025   0.21812   0.02780  -0.14776
PHI AT LEADING EDGE = -0.250165

  51   0.99743   0.69509   1.28466  11.37122   0.14918 -44.01260  -5.44929  -0.00006   0.21928   0.02780   0.14786
  52   0.98786   0.68742   1.29055   9.96620   0.14918 -23.78140  -3.48611   0.00001   0.18677   0.02892   0.14786
  53   0.96987   0.67311   1.30172  10.56836   0.14918 -15.22624  -2.64247   0.00008   0.14570   0.03015   0.14786
  54   0.94421   0.65287   1.31783  10.95838   0.14918  -9.61764  -2.20628  -0.00001   0.10289   0.03102   0.14786
  55   0.91150   0.62729   1.33859  10.96495   0.14918  -5.04471  -1.91955  -0.00010   0.06077   0.03127   0.14786
  56   0.87238   0.59698   1.36367  10.75518   0.14918  -1.09158  -1.69637  -0.00004   0.02095   0.03077   0.14786
  57   0.82756   0.56256   1.39273  10.45991   0.14918   2.30453  -1.50975  -0.00008  -0.01534   0.02945   0.14786
  58   0.77778   0.52471   1.42535  10.14643   0.14918   5.13865  -1.34864  -0.00010  -0.04718   0.02728   0.14786
  59   0.72385   0.48411   1.46109   9.84451   0.14918   7.39946  -1.20657  -0.00004  -0.07391   0.02428   0.14786
  60   0.66661   0.44145   1.49946   9.56763   0.14918   9.08279  -1.07795  -0.00002  -0.09506   0.02050   0.14786
  61   0.60695   0.39746   1.53990   9.32081   0.14918  10.18887  -0.95933  -0.00006  -0.11035   0.01601   0.14786
  62   0.54577   0.35287   1.58187   9.10561   0.14918  10.72805  -0.84685  -0.00009  -0.11965   0.01092   0.14786
  63   0.48401   0.30839   1.62476   8.92147   0.14918  10.71595  -0.73769  -0.00004  -0.12291   0.00534   0.14786
  64   0.42259   0.26472   1.66797   8.76692   0.14918  10.17482  -0.62870  -0.00006  -0.12021  -0.00059   0.14786
  65   0.36244   0.22255   1.71086   8.64020   0.14918   9.13097  -0.51648  -0.00004  -0.11170  -0.00672   0.14786
  66   0.30445   0.18251   1.75282   8.54068   0.14918   7.61369  -0.39643  -0.00008  -0.09756  -0.01292   0.14786
  67   0.24948   0.14523   1.79323   8.46711   0.14918   5.65260  -0.26413  -0.00003  -0.07806  -0.01903   0.14786
  68   0.19836   0.11124   1.83149   8.41829   0.14918   3.27658  -0.11194  -0.00007  -0.05352  -0.02491   0.14786
  69   0.15184   0.08106   1.86703   8.39436   0.14918   0.50531   0.07140  -0.00007  -0.02430  -0.03042   0.14786
  70   0.11061   0.05511   1.89931   8.39488   0.14918  -2.66272   0.30314  -0.00006   0.00921  -0.03543   0.14786
  71   0.07527   0.03376   1.92784   8.41893   0.14918  -6.29187   0.61714  -0.00006   0.04655  -0.03984   0.14786
  72   0.04635   0.01731   1.95217   8.46179   0.14918 -10.63195   1.07910  -0.00003   0.08729  -0.04355   0.14786
  73   0.02428   0.00596   1.97192   8.49455   0.14918 -16.61971   1.84791   0.00002   0.13106  -0.04650   0.14786
  74   0.00939  -0.00014   1.98677   8.32860   0.14918 -28.49463   3.37257  -0.00001   0.17789  -0.04862   0.14786
  75   0.00188  -0.00093   1.99647   5.95462   0.14918 -65.71062   6.67641  -0.00026   0.22708  -0.04962   0.14786
  76   0.00188   0.00356   2.00083  -4.45550   0.14918 -80.42683   7.43525   0.00071   0.27719  -0.04961   0.14786
  77   0.00939   0.01323   1.99977  -7.57422   0.14918 -51.30396   4.71352  -0.00001   0.32749  -0.04859   0.14786
  78   0.02428   0.02790   1.99326  -8.03390   0.14918 -43.77545   3.30518  -0.00002   0.37667  -0.04647   0.14786
  79   0.04635   0.04732   1.98136  -8.14580   0.14918 -42.98891   2.57176   0.00003   0.42366  -0.04352   0.14786
  80   0.07527   0.07119   1.96424  -8.19124   0.14918 -44.16951   2.12357   0.00002   0.46842  -0.03983   0.14786
  81   0.11061   0.09911   1.94210  -8.22886   0.14918 -45.97194   1.81527   0.00009   0.51049  -0.03547   0.14786
  82   0.15184   0.13066   1.91526  -8.27457   0.14918 -47.90329   1.58543   0.00006   0.54936  -0.03052   0.14786
  83   0.19836   0.16534   1.88409  -8.33581   0.14918 -49.73038   1.40202   0.00005   0.58446  -0.02510   0.14786
  84   0.24948   0.20264   1.84905  -8.41553   0.14918 -51.32961   1.24865   0.00007   0.61531  -0.01933   0.14786
  85   0.30445   0.24199   1.81066  -8.51566   0.14918 -52.62383   1.11475   0.00002   0.64142  -0.01334   0.14786
  86   0.36244   0.28281   1.76947  -8.63885   0.14918 -53.56149   0.99303   0.00005   0.66240  -0.00729   0.14786
  87   0.42259   0.32451   1.72611  -8.78655   0.14918 -54.11133   0.87951   0.00002   0.67792  -0.00131   0.14786
  88   0.48400   0.36648   1.68126  -8.96037   0.14918 -54.25262   0.77008   0.00003   0.68771   0.00446   0.14786
  89   0.54577   0.40811   1.63559  -9.16242   0.14918 -53.97629   0.66172   0.00003   0.69161   0.00988   0.14786
  90   0.60694   0.44881   1.58985  -9.39461   0.14918 -53.28275   0.55166   0.00002   0.68954   0.01481   0.14786
  91   0.66661   0.48802   1.54475  -9.65748   0.14918 -52.18385   0.43771   0.00003   0.68156   0.01916   0.14786
  92   0.72385   0.52518   1.50103  -9.94989   0.14918 -50.70218   0.31680   0.00005   0.66783   0.02281   0.14786
  93   0.77778   0.55977   1.45944 -10.26650   0.14918 -48.87601   0.18732   0.00009   0.64871   0.02570   0.14786
  94   0.82756   0.59133   1.42070 -10.59347   0.14918 -46.76427   0.04775   0.00009   0.62480   0.02777   0.14786
  95   0.87238   0.61941   1.38549 -10.89909   0.14918 -44.45845  -0.09690   0.00001   0.59714   0.02902   0.14786
  96   0.91150   0.64364   1.35449 -11.11176   0.14918 -42.10207  -0.22406   0.00012   0.56762   0.02947   0.14786
  97   0.94421   0.66367   1.32834 -11.09540   0.14918 -39.92393  -0.25775   0.00011   0.53990   0.02918   0.14786
  98   0.96987   0.67921   1.30765 -10.70689   0.14918 -38.37997   0.00495  -0.00002   0.52054   0.02829   0.14786
  99   0.98786   0.68998   1.29303 -10.27522   0.14918 -38.92798   0.87120   0.00006   0.51667   0.02705   0.14786
 100   0.99743   0.69565   1.28520 -12.24565   0.14918 -44.37007   2.33469   0.00048   0.52493   0.02594   0.14786
PHI AT LEADING EDGE = -0.252055


AIRFOIL NO    1
   CD = 36.586426    CL=-36.901199    CM= 25.629196


AIRFOIL NO    2
   CD = 36.588806    CL= 36.907700    CM=-25.633362


TRAILING VORTICES DATA

   M    X1I(M)     Y1I(M)      X1(M)       Y1(M)     CIRC1      X2I(M)     Y2I(M)     X2(M)       Y2(M)       CIRC2

   1   1.001514  -0.019158   0.711955   0.704640   0.303002   1.001513   0.019157   0.711953   1.295360  -0.303004



TIME STEP TK =   0.325000          TK - TKM1 =   0.025000


ALPHA(1)  = -45.800003      ALPHA(2)  =  45.800003
```

```
OMEGA(1)  =   0.000000      OMEGA(2)  =   0.000000
U(1)      =   0.000000       U(2)     =   0.000000
V(1)      =   0.000000       V(2)     =   0.000000


 NITR   VXW(1)     VYW(1)    WAKE(1)  THETA(1)      GAMK(1)        VXW(2)     VYW(2)    WAKE(2)    THETA(2)   GAMK(2)

   0  1.140368 -0.308443  0.029534 -0.264156 -0.543751E+00  1.140563  0.308584  0.029539  0.264228  0.543745E+00
   1  1.112525 -0.287883  0.028729 -0.253211 -0.561764E+00  1.112760  0.288016  0.028736  0.253272  0.561758E+00
   2  1.111856 -0.297002  0.028771 -0.261028 -0.561607E+00  1.112068  0.297086  0.028777  0.261051  0.561601E+00
   3  1.110073 -0.295391  0.028718 -0.260074 -0.561636E+00  1.110258  0.295544  0.028723  0.260161  0.561624E+00
   4  1.110075 -0.296067  0.028722 -0.260642 -0.561621E+00  1.110245  0.296232  0.028727  0.260742  0.561607E+00


CONVERGED SOLUTION OBTAINED AFTER NITR =   4

  J    XI(J)     X(J)      Y(J)      Q(J)      GAMMA     CP(J)      V(J)      VN(J)      PHIK       PHI      INTGAMK

  1   0.99743   0.69565   0.71479  -8.31079  -0.56162  -0.75150  -0.43606  -0.00311   1.03243   1.01291  -0.56152
  2   0.98786   0.68998   0.70697  -7.88176  -0.56162  -0.88682  -0.56119   0.00014   1.03279   1.01314  -0.56152
  3   0.96987   0.67921   0.69235  -7.63788  -0.56162  -1.00762  -0.64537   0.00031   1.03210   1.01221  -0.56152
  4   0.94421   0.66367   0.67166  -7.47783  -0.56162  -1.10478  -0.70165   0.00020   1.02973   1.00957  -0.56152
  5   0.91150   0.64364   0.64551  -7.37372  -0.56162  -1.16931  -0.73690   0.00020   1.02566   1.00533  -0.56152
  6   0.87238   0.61941   0.61451  -7.31505  -0.56162  -1.20341  -0.76130   0.00015   1.02012   0.9998^  -0.56152
  7   0.82756   0.59133   0.57930  -7.29550  -0.56162  -1.21578  -0.78391  -0.00018   1.01324   0.99322  -0.56152
  8   0.77778   0.55977   0.54055  -7.30553  -0.56162  -1.21711  -0.81085   0.00003   1.00491   0.98541  -0.56152
  9   0.72385   0.52518   0.49897  -7.33819  -0.56162  -1.21787  -0.84540   0.00002   0.99478   0.97599  -0.56152
 10   0.66661   0.48802   0.45526  -7.38680  -0.56162  -1.22874  -0.88971   0.00002   0.98235   0.96439  -0.56152
 11   0.60695   0.44882   0.41016  -7.44796  -0.56162  -1.25672  -0.94368   0.00003   0.96705   0.94998  -0.56152
 12   0.54577   0.40811   0.36441  -7.51981  -0.56162  -1.30895  -1.00757   0.00003   0.94838   0.93221  -0.56152
 13   0.48401   0.36648   0.31875  -7.60185  -0.56162  -1.39115  -1.08124   0.00003   0.92592   0.91064  -0.56152
 14   0.42259   0.32451   0.27389  -7.69388  -0.56162  -1.51066  -1.16544   0.00003   0.89940   0.88500  -0.56152
 15   0.36244   0.28281   0.23053  -7.79605  -0.56162  -1.67740  -1.26175   0.00002   0.86869   0.85512  -0.56152
 16   0.30445   0.24199   0.18935  -7.90916  -0.56162  -1.90711  -1.37333   0.00003   0.83376   0.82100  -0.56152
 17   0.24948   0.20264   0.15095  -8.03430  -0.56162  -2.22107  -1.50389   0.00003   0.79473   0.78274  -0.56152
 18   0.19836   0.16534   0.11591  -8.17119  -0.56162  -2.65888  -1.66094   0.00003   0.75181   0.74056  -0.56152
 19   0.15184   0.13066   0.08474  -8.32006  -0.56162  -3.28859  -1.85625   0.00002   0.70530   0.69476  -0.56152
 20   0.11061   0.09911   0.05790  -8.47990  -0.56162  -4.23418  -2.10872   0.00002   0.65558   0.64574  -0.56152
 21   0.07527   0.07119   0.03576  -8.64449  -0.56162  -5.76310  -2.45555   0.00003   0.60306   0.59390  -0.56152
 22   0.04635   0.04732   0.01864  -8.79769  -0.56162  -8.49495  -2.96896   0.00004   0.54818   0.53968  -0.56152
```

```
  23   0.02428   0.02790   0.00674  -8.87481  -0.56162 -14.22014  -3.82003   0.00005   0.49126   0.48342  -0.56152
  24   0.00939   0.01323   0.00023  -8.55794  -0.56162 -29.46707  -5.46741   0.00005   0.43221   0.42503  -0.56152
  25   0.00188   0.00356  -0.00083  -5.18752  -0.56162 -74.83887  -8.67857   0.00008   0.37213   0.36561  -0.56152
  26   0.00188  -0.00093   0.00353   6.67230  -0.56162 -62.30748  -7.92712  -0.00001   0.31235   0.30649  -0.56152
  27   0.00939  -0.00014   0.01323   9.30367  -0.56162 -16.56010  -4.14077  -0.00001   0.25327   0.24809  -0.56152
  28   0.02428   0.00596   0.02808   9.33153  -0.56162  -5.00980  -2.37712   0.00000   0.19635   0.19186  -0.56152
  29   0.04635   0.01731   0.04783   9.11115  -0.56162  -1.52558  -1.49071   0.00000   0.14235   0.13855  -0.56152
  30   0.07527   0.03376   0.07216   8.87076  -0.56162  -0.17362  -0.96302   0.00000   0.09109   0.08802  -0.56152
  31   0.11061   0.05511   0.10069   8.64545  -0.56162   0.44127  -0.60961   0.00001   0.04287   0.04053  -0.56152
  32   0.15184   0.08106   0.13297   8.44034  -0.56162   0.74913  -0.35385   0.00002  -0.00196  -0.00353  -0.56152
  33   0.19836   0.11124   0.16851   8.25513  -0.56162   0.91398  -0.15653   0.00001  -0.04302  -0.04379  -0.56152
  34   0.24948   0.14523   0.20677   8.08841  -0.56162   1.00566   0.00240   0.00001  -0.07999  -0.07992  -0.56152
  35   0.30445   0.18251   0.24718   7.93796  -0.56162   1.05779   0.13571   0.00001  -0.11257  -0.11162  -0.56152
  36   0.36244   0.22255   0.28914   7.80264  -0.56162   1.08699   0.25185   0.00001  -0.14053  -0.13865  -0.56152
  37   0.42259   0.26472   0.33203   7.68117  -0.56162   1.10203   0.35567   0.00001  -0.16368  -0.16082  -0.56152
  38   0.48400   0.30839   0.37523   7.57179  -0.56162   1.10662   0.45177   0.00002  -0.18186  -0.17798  -0.56152
  39   0.54577   0.35287   0.41812   7.47382  -0.56162   1.10177   0.54334   0.00003  -0.19500  -0.19003  -0.56152
  40   0.60694   0.39746   0.46009   7.38684  -0.56162   1.08640   0.63307   0.00003  -0.20306  -0.19697  -0.56152
  41   0.66661   0.44145   0.50054   7.31075  -0.56162   1.05820   0.72300   0.00003  -0.20611  -0.19885  -0.56152
  42   0.72385   0.48411   0.53890   7.24604  -0.56162   1.01234   0.81559   0.00002  -0.20431  -0.19585  -0.56152
  43   0.77778   0.52471   0.57465   7.19472  -0.56162   0.94412   0.91209  -0.00016  -0.19799  -0.18829  -0.56152
  44   0.82756   0.56256   0.60727   7.16136  -0.56162   0.84605   1.01437  -0.00020  -0.18765  -0.17671  -0.56152
  45   0.87238   0.59698   0.63633   7.15340  -0.56162   0.71112   1.12314   0.00012  -0.17401  -0.16186  -0.56152
  46   0.91150   0.62729   0.66141   7.17789  -0.56162   0.53270   1.23833   0.00022  -0.15812  -0.14479  -0.56152
  47   0.94421   0.65287   0.68217   7.24350  -0.56162   0.30551   1.35901   0.00019  -0.14131  -0.12690  -0.56152
  48   0.96987   0.67311   0.69828   7.36447  -0.56162   0.02290   1.48466   0.00035  -0.12526  -0.10992  -0.56152
  49   0.98786   0.68742   0.70945   7.57407  -0.56162  -0.31961   1.61384   0.00040  -0.11196  -0.09590  -0.56152
  50   0.99743   0.69509   0.71534   7.97046  -0.56162  -0.76023   1.75469  -0.00245  -0.10383  -0.08735  -0.56152
PHI AT LEADING EDGE = -0.342170

  51   0.99743   0.69509   1.28466   7.97778   0.56161  -0.76037  -1.75452   0.00037  -0.10290  -0.08642   0.56158
  52   0.98786   0.68742   1.29055   7.57579   0.56161  -0.31826  -1.61323   0.00002  -0.11101  -0.09496   0.56158
  53   0.96987   0.67311   1.30172   7.36531   0.56161   0.02290  -1.48450   0.00002  -0.12428  -0.10895   0.56158
  54   0.94421   0.65287   1.31783   7.24414   0.56161   0.30521  -1.35899  -0.00005  -0.14032  -0.12592   0.56158
  55   0.91150   0.62729   1.33859   7.17822   0.56161   0.53234  -1.23837  -0.00008  -0.15711  -0.14370   0.56158
  56   0.87238   0.59698   1.36367   7.15375   0.56161   0.71057  -1.12333  -0.00002  -0.17299  -0.16083   0.56158
  57   0.82756   0.56256   1.39273   7.16211   0.56161   0.84578  -1.01446  -0.00004  -0.18660  -0.17567   0.56158
  58   0.77778   0.52471   1.42535   7.19531   0.56161   0.94412  -0.91204  -0.00007  -0.19692  -0.18722   0.56158
  59   0.72385   0.48411   1.46109   7.24624   0.56161   1.01239  -0.81549  -0.00002  -0.20321  -0.19474   0.56158
  60   0.66661   0.44145   1.49946   7.31086   0.56161   1.05811  -0.72296   0.00000  -0.20496  -0.19770   0.56158
```

61	0.60695	0.39746	1.53990	7.38686	0.56161	1.08626	-0.63304	-0.00004	-0.20188	-0.19579	0.56158
62	0.54577	0.35287	1.58187	7.47380	0.56161	1.10157	-0.54333	-0.00007	-0.19378	-0.18882	0.56158
63	0.48401	0.30839	1.62476	7.57176	0.56161	1.10640	-0.45177	-0.00003	-0.18061	-0.17673	0.56158
64	0.42259	0.26472	1.66797	7.68113	0.56161	1.10179	-0.35568	-0.00004	-0.16239	-0.15954	0.56158
65	0.36244	0.22255	1.71086	7.80257	0.56161	1.08673	-0.25187	-0.00003	-0.13921	-0.13733	0.56158
66	0.30445	0.18251	1.75282	7.93788	0.56161	1.05751	-0.13573	-0.00006	-0.11122	-0.11027	0.56158
67	0.24948	0.14523	1.79323	8.08831	0.56161	1.00537	-0.00242	-0.00003	-0.07859	-0.07853	0.56158
68	0.19836	0.11124	1.83149	8.25502	0.56161	0.91369	0.15650	-0.00007	-0.04159	-0.04237	0.56158
69	0.15184	0.08106	1.86703	8.44022	0.56161	0.74885	0.35382	-0.00006	-0.00050	-0.00207	0.56158
70	0.11061	0.05511	1.89931	8.64535	0.56161	0.44097	0.60960	-0.00006	0.04435	0.04201	0.56158
71	0.07527	0.03376	1.92784	8.87064	0.56161	-0.17392	0.96300	-0.00006	0.09260	0.08952	0.56158
72	0.04635	0.01731	1.95217	9.11104	0.56161	-1.52584	1.49067	-0.00003	0.14387	0.14008	0.56158
73	0.02428	0.00596	1.97192	9.33131	0.56161	-5.01016	2.37712	0.00002	0.19789	0.19340	0.56158
74	0.00939	-0.00014	1.98677	9.30412	0.56161	-16.55652	4.14030	-0.00003	0.25482	0.24964	0.56158
75	0.00188	-0.00093	1.99647	6.67174	0.56161	-62.30864	7.92717	-0.00029	0.31391	0.30805	0.56158
76	0.00188	0.00356	2.00083	-5.18778	0.56161	-74.83470	8.67830	0.00082	0.37369	0.36717	0.56158
77	0.00939	0.01323	1.99977	-8.55713	0.56161	-29.47173	5.46780	-0.00002	0.43377	0.42658	0.56158
78	0.02428	0.02790	1.99326	-8.87498	0.56161	-14.22013	3.81998	-0.00002	0.49282	0.48497	0.56158
79	0.04635	0.04732	1.98136	-8.79768	0.56161	-8.49520	2.96894	0.00004	0.54973	0.54122	0.56158
80	0.07527	0.07119	1.96424	-8.64440	0.56161	-5.76331	2.45553	0.00002	0.60459	0.59542	0.56158
81	0.11061	0.09911	1.94210	-8.47980	0.56161	-4.23444	2.10871	0.00009	0.65709	0.64724	0.56158
82	0.15184	0.13066	1.91526	-8.31996	0.56161	-3.28885	1.85625	0.00005	0.70679	0.69625	0.56158
83	0.19836	0.16534	1.88409	-8.17109	0.56161	-2.65909	1.66094	0.00005	0.75327	0.74202	0.56158
84	0.24948	0.20264	1.84905	-8.03420	0.56161	-2.22129	1.50387	0.00006	0.79617	0.78417	0.56158
85	0.30445	0.24199	1.81066	-7.90908	0.56161	-1.90733	1.37332	0.00001	0.83517	0.82240	0.56158
86	0.36244	0.28281	1.76947	-7.79598	0.56161	-1.67757	1.26174	0.00005	0.87007	0.85650	0.56158
87	0.42259	0.32451	1.72611	-7.69383	0.56161	-1.51081	1.16542	0.00001	0.90076	0.88635	0.56158
88	0.48400	0.36648	1.68126	-7.60182	0.56161	-1.39129	1.08122	0.00002	0.92724	0.91196	0.56158
89	0.54577	0.40811	1.63559	-7.51979	0.56161	-1.30909	1.00754	0.00002	0.94967	0.93350	0.56158
90	0.60694	0.44881	1.58985	-7.44801	0.56161	-1.25688	0.94366	0.00001	0.96832	0.95124	0.56158
91	0.66661	0.48802	1.54475	-7.38693	0.56161	-1.22889	0.88969	-0.00001	0.98359	0.96562	0.56158
92	0.72385	0.52518	1.50103	-7.33839	0.56161	-1.21796	0.84533	0.00003	0.99599	0.97720	0.56158
93	0.77778	0.55977	1.45944	-7.30589	0.56161	-1.21710	0.81077	0.00005	1.00609	0.98659	0.56158
94	0.82756	0.59133	1.42070	-7.29552	0.56161	-1.21590	0.78394	0.00005	1.01440	0.99438	0.56158
95	0.87238	0.61941	1.38549	-7.31573	0.56161	-1.20364	0.76140	-0.00001	1.02127	1.00097	0.56158
96	0.91150	0.64364	1.35449	-7.37451	0.56161	-1.16933	0.73687	0.00006	1.02679	1.00646	0.56158
97	0.94421	0.66367	1.32834	-7.47881	0.56161	-1.10479	0.70149	0.00006	1.03084	1.01068	0.56158
98	0.96987	0.67921	1.30765	-7.63937	0.56161	-1.00745	0.64501	-0.00002	1.03320	1.01331	0.56158
99	0.98786	0.68998	1.29303	-7.88405	0.56161	-0.88592	0.56011	-0.00004	1.03387	1.01422	0.56158
100	0.99743	0.69565	1.28520	-8.31079	0.56161	-0.75051	0.43483	0.00065	1.03350	1.01398	0.56158

```
PHI AT LEADING EDGE = -0.343733



AIRFOIL NO    1
   CD =  1.243259    CL= -3.090490    CM=  1.118548


AIRFOIL NO    2
   CD =  1.243220    CL=  3.090449    CM= -1.118505


TRAILING VORTICES DATA

   M    X1I(M)     Y1I(M)     X1(M)      Y1(M)      CIRC1      X2I(M)     Y2I(M)     X2(M)      Y2(M)      CIRC2

   1  1.187005  -0.176652  0.954182   0.727821   0.303002   1.187003   0.176660   0.954187   1.272185  -0.303004
   2  1.095049  -0.208717  0.913061   0.639542   0.188150   1.095038   0.208701   0.913042   1.360454  -0.188153
   3  1.121147  -0.303922  0.999510   0.591878   0.124051   1.121119   0.303913   0.999484   1.408134  -0.124049
   4  1.191340  -0.323759  1.062667   0.628371   0.091023   1.191307   0.323764   1.062647   1.371655  -0.091020
   5  1.237700  -0.292511  1.072584   0.683392   0.071417   1.237677   0.292530   1.072582   1.316638  -0.071417
   6  1.259034  -0.245758  1.053941   0.731281   0.058907   1.259017   0.245782   1.053946   1.268747  -0.058903
   7  1.262055  -0.177477  1.007095   0.781050   0.051384   1.262053   0.177504   1.007113   1.218970  -0.051380
   8  1.214726  -0.099646  0.918302   0.801381   0.047029   1.214730   0.099660   0.918315   1.198627  -0.047022
   9  1.155381  -0.062438  0.850254   0.784775   0.044248   1.155385   0.062449   0.850264   1.215229  -0.044245
  10  1.110906  -0.040555  0.803559   0.768147   0.042095   1.110907   0.040565   0.803567   1.231859  -0.042090
  11  1.074797  -0.024334  0.766756   0.753568   0.040097   1.074800   0.024345   0.766765   1.246436  -0.040094
  12  1.042765  -0.011568  0.735272   0.739505   0.038090   1.042772   0.011574   0.735282   1.260494  -0.038101
  13  1.013875  -0.003701  0.709491   0.724278   0.036077   1.013878   0.003703   0.709495   1.275722  -0.036063



TIME STEP TK =   0.649999          TK - TKM1 =   0.025000


ALPHA(1)  = -45.800003      ALPHA(2)  =  45.800003
OMEGA(1)  =   0.000000      OMEGA(2)  =   0.000000
U(1)      =   0.000000       U(2)     =   0.000000
```

```
V(1)      =   0.000000       V(2)      =   0.000000

 NITR   VXW(1)     VYW(1)    WAKE(1)  THETA(1)      GAMK(1)         VXW(2)     VYW(2)    WAKE(2)    THETA(2)    GAMK(2)

   0  0.839121 -0.213523  0.021647 -0.249173 -0.720169E+00  0.838787  0.213520  0.021638  0.249264  0.720154E+00
   1  0.824820 -0.204294  0.021244 -0.242797 -0.729964E+00  0.824530  0.204175  0.021236  0.242743  0.729952E+00
   2  0.824400 -0.208782  0.021261 -0.248039 -0.729913E+00  0.824274  0.208841  0.021258  0.248142  0.729891E+00
   3  0.823473 -0.208012  0.021233 -0.247427 -0.729926E+00  0.823316  0.207992  0.021230  0.247450  0.729907E+00
   4  0.823489 -0.208489  0.021237 -0.247968 -0.729913E+00  0.823363  0.208204  0.021232  0.247678  0.729911E+00
   5  0.823380 -0.208250  0.021233 -0.247726 -0.729923E+00  0.823355  0.207990  0.021230  0.247437  0.729924E+00
   6  0.823445 -0.208370  0.021235 -0.247844 -0.729920E+00  0.823353  0.208299  0.021232  0.247790  0.729904E+00


CONVERGED SOLUTION OBTAINED AFTER NITR =    6

  J    XI(J)      X(J)       Y(J)      Q(J)       GAMMA     CP(J)        V(J)       VN(J)      PHIK       PHI      INTGAMK

  1   0.99743   0.69565   0.71479  -8.17944  -0.72991   0.18394  -0.33491   0.00097   1.19777   1.18897  -0.72992
  2   0.98786   0.68998   0.70697  -7.91132  -0.72991   0.09351  -0.44637   0.00005   1.19929   1.19045  -0.72992
  3   0.96987   0.67921   0.69235  -7.77254  -0.72991   0.00115  -0.53382   0.00010   1.20079   1.19186  -0.72992
  4   0.94421   0.66367   0.67166  -7.68732  -0.72991  -0.09385  -0.60888   0.00008   1.20119   1.19215  -0.72992
  5   0.91150   0.64364   0.64551  -7.63164  -0.72991  -0.18923  -0.67520   0.00010   1.19976   1.19059  -0.72992
  6   0.87238   0.61941   0.61451  -7.59746  -0.72991  -0.28522  -0.73675   0.00009   1.19595   1.18667  -0.72992
  7   0.82756   0.59133   0.57930  -7.58434  -0.72991  -0.38176  -0.79586   0.00006   1.18935   1.18000  -0.72992
  8   0.77778   0.55977   0.54055  -7.59203  -0.72991  -0.47881  -0.85403   0.00021   1.17961   1.17024  -0.72992
  9   0.72385   0.52518   0.49897  -7.62047  -0.72991  -0.57881  -0.91296   0.00002   1.16646   1.15714  -0.72992
 10   0.66661   0.48802   0.45526  -7.66767  -0.72991  -0.68628  -0.97494   0.00003   1.14963   1.14044  -0.72992
 11   0.60695   0.44882   0.41016  -7.73355  -0.72991  -0.80502  -1.04103   0.00004   1.12891   1.11989  -0.72992
 12   0.54577   0.40811   0.36441  -7.81743  -0.72991  -0.94180  -1.11317   0.00003   1.10409   1.09531  -0.72992
 13   0.48401   0.36648   0.31875  -7.91896  -0.72991  -1.10318  -1.19271   0.00002   1.07504   1.06653  -0.72992
 14   0.42259   0.32451   0.27389  -8.03749  -0.72991  -1.29878  -1.28166   0.00002   1.04168   1.03348  -0.72992
 15   0.36244   0.28281   0.23053  -8.17255  -0.72991  -1.54135  -1.38264   0.00002   1.00403   0.99616  -0.72992
 16   0.30445   0.24199   0.18935  -8.32434  -0.72991  -1.85113  -1.49972   0.00002   0.96215   0.95462  -0.72992
 17   0.24948   0.20264   0.15095  -8.49350  -0.72991  -2.25455  -1.63744   0.00002   0.91620   0.90904  -0.72992
 18   0.19836   0.16534   0.11591  -8.67929  -0.72991  -2.79994  -1.80440   0.00001   0.86644   0.85964  -0.72992
 19   0.15184   0.13066   0.08474  -8.88170  -0.72991  -3.56941  -2.01370   0.00002   0.81317   0.80674  -0.72992
 20   0.11061   0.09911   0.05790  -9.09955  -0.72991  -4.71237  -2.28648   0.00002   0.75675   0.75069  -0.72992
 21   0.07527   0.07119   0.03576  -9.32625  -0.72991  -6.55085  -2.66398   0.00002   0.69758   0.69191  -0.72992
 22   0.04635   0.04732   0.01864  -9.54449  -0.72991  -9.83291  -3.22638   0.00004   0.63608   0.63079  -0.72992
```

```
  23  0.02428   0.02790   0.00674  -9.68460  -0.72991 -16.72728  -4.16355   0.00004   0.57252   0.56761  -0.72992
  24  0.00939   0.01323   0.00023  -9.40109  -0.72991 -35.18681  -5.98554   0.00004   0.50670   0.50220  -0.72992
  25  0.00188   0.00356  -0.00083  -5.77941  -0.72991 -90.71704  -9.55979   0.00007   0.43979   0.43569  -0.72992
  26  0.00188  -0.00093   0.00353   7.23625  -0.72991 -77.12851  -8.82237  -0.00002   0.37312   0.36944  -0.72992
  27  0.00939  -0.00014   0.01323  10.13224  -0.72991 -21.20012  -4.68402  -0.00001   0.30699   0.30374  -0.72992
  28  0.02428   0.00596   0.02808  10.13186  -0.72991  -6.77924  -2.74849   0.00000   0.24287   0.24006  -0.72992
  29  0.04635   0.01731   0.04783   9.85076  -0.72991  -2.34912  -1.77759   0.00000   0.18157   0.17920  -0.72992
  30  0.07527   0.03376   0.07216   9.54653  -0.72991  -0.59843  -1.20251   0.00000   0.12280   0.12089  -0.72992
  31  0.11061   0.05511   0.10069   9.25986  -0.72991   0.21301  -0.82031   0.00001   0.06678   0.06535  -0.72992
  32  0.15184   0.08106   0.13297   8.99712  -0.72991   0.62721  -0.54657   0.00001   0.01384   0.01291  -0.72992
  33  0.19836   0.11124   0.16851   8.75855  -0.72991   0.85349  -0.33819   0.00000  -0.03571  -0.03611  -0.72992
  34  0.24948   0.14523   0.20677   8.54297  -0.72991   0.98192  -0.17312   0.00002  -0.08157  -0.08142  -0.72992
  35  0.30445   0.18251   0.24718   8.34836  -0.72991   1.05683  -0.03740   0.00000  -0.12351  -0.12278  -0.72992
  36  0.36244   0.22255   0.28914   8.17409  -0.72991   1.10098   0.07809   0.00001  -0.16132  -0.15998  -0.72992
  37  0.42259   0.26472   0.33203   8.01927  -0.72991   1.12671   0.17855   0.00001  -0.19483  -0.19284  -0.72992
  38  0.48400   0.30839   0.37523   7.88277  -0.72991   1.14055   0.26881   0.00002  -0.22392  -0.22126  -0.72992
  39  0.54577   0.35287   0.41812   7.76455  -0.72991   1.14575   0.35212   0.00002  -0.24850  -0.24513  -0.72992
  40  0.60694   0.39746   0.46009   7.66482  -0.72991   1.14334   0.43119   0.00003  -0.26853  -0.26442  -0.72992
  41  0.66661   0.44145   0.50054   7.58328  -0.72991   1.13323   0.50785  -0.00009  -0.28402  -0.27913  -0.72992
  42  0.72385   0.48411   0.53890   7.52032  -0.72991   1.11342   0.58440  -0.00009  -0.29505  -0.28936  -0.72992
  43  0.77778   0.52471   0.57465   7.47589  -0.72991   1.08156   0.66198  -0.00011  -0.30178  -0.29528  -0.72992
  44  0.82756   0.56256   0.60727   7.45023  -0.72991   1.03367   0.74228  -0.00011  -0.30451  -0.29720  -0.72992
  45  0.87238   0.59698   0.63633   7.44485  -0.72991   0.96644   0.82574   0.00016  -0.30369  -0.29559  -0.72992
  46  0.91150   0.62729   0.66141   7.45912  -0.72991   0.87617   0.91246   0.00029  -0.30000  -0.29114  -0.72992
  47  0.94421   0.65287   0.68217   7.49373  -0.72991   0.75901   1.00233   0.00022  -0.29430  -0.28475  -0.72992
  48  0.96987   0.67311   0.69828   7.55789  -0.72991   0.60859   1.09624   0.00052  -0.28771  -0.27758  -0.72992
  49  0.98786   0.68742   0.70945   7.67645  -0.72991   0.42166   1.19328   0.00004  -0.28159  -0.27102  -0.72992
  50  0.99743   0.69509   0.71534   7.93091  -0.72991   0.17458   1.30042   0.00073  -0.27756  -0.26674  -0.72992
PHI AT LEADING EDGE = -0.406411

  51  0.99743   0.69509   1.28466   7.91406   0.72991   0.18208  -1.29858  -0.00021  -0.27656  -0.26571   0.73001
  52  0.98786   0.68742   1.29055   7.67178   0.72991   0.42557  -1.19255   0.00043  -0.28058  -0.26998   0.73001
  53  0.96987   0.67311   1.30172   7.55367   0.72991   0.61129  -1.09577  -0.00001  -0.28668  -0.27653   0.73001
  54  0.94421   0.65287   1.31783   7.49132   0.72991   0.76064  -1.00219  -0.00005  -0.29325  -0.28368   0.73001
  55  0.91150   0.62729   1.33859   7.45734   0.72991   0.87737  -0.91244  -0.00012  -0.29892  -0.29005   0.73001
  56  0.87238   0.59698   1.36367   7.44392   0.72991   0.96731  -0.82587  -0.00004  -0.30261  -0.29449   0.73001
  57  0.82756   0.56256   1.39273   7.45010   0.72991   1.03444  -0.74240  -0.00009  -0.30341  -0.29609   0.73001
  58  0.77778   0.52471   1.42535   7.47598   0.72991   1.08217  -0.66205  -0.00007  -0.30065  -0.29415   0.73001
  59  0.72385   0.48411   1.46109   7.52040   0.72991   1.11394  -0.58439  -0.00002  -0.29389  -0.28820   0.73001
  60  0.66661   0.44145   1.49946   7.58330   0.72991   1.13372  -0.50780  -0.00002  -0.28283  -0.27794   0.73001
```

61	0.60695	0.39746	1.53990	7.66453	0.72991	1.14372	-0.43113	-0.00006	-0.26730	-0.26318	0.73001
62	0.54577	0.35287	1.58187	7.76428	0.72991	1.14596	-0.35212	-0.00008	-0.24723	-0.24386	0.73001
63	0.48401	0.30839	1.62476	7.88249	0.72991	1.14070	-0.26882	-0.00004	-0.22261	-0.21995	0.73001
64	0.42259	0.26472	1.66797	8.01901	0.72991	1.12681	-0.17856	-0.00005	-0.19348	-0.19150	0.73001
65	0.36244	0.22255	1.71086	8.17382	0.72991	1.10107	-0.07812	-0.00004	-0.15994	-0.15860	0.73001
66	0.30445	0.18251	1.75282	8.34813	0.72991	1.05690	0.03738	-0.00006	-0.12210	-0.1213,	0.73001
67	0.24948	0.14523	1.79323	8.54271	0.72991	0.98196	0.17309	-0.00003	-0.08012	-0.07997	0.73001
68	0.19836	0.11124	1.83149	8.75831	0.72991	0.85350	0.33817	-0.00007	-0.03422	-0.03462	0.73001
69	0.15184	0.08106	1.86703	8.99688	0.72991	0.62718	0.54655	-0.00006	0.01536	0.01444	0.73001
70	0.11061	0.05511	1.89931	9.25964	0.72991	0.21295	0.82030	-0.00006	0.06833	0.06690	0.73001
71	0.07527	0.03376	1.92784	9.54632	0.72991	-0.59847	1.20248	-0.00006	0.12437	0.12247	0.73001
72	0.04635	0.01731	1.95217	9.85058	0.72991	-2.34903	1.77753	-0.00003	0.18317	0.18080	0.73001
73	0.02428	0.00596	1.97192	10.13156	0.72991	-6.77941	2.74850	0.00002	0.24449	0.24167	0.73001
74	0.00939	-0.00014	1.98677	10.13269	0.72991	-21.19522	4.68348	-0.00004	0.30861	0.30536	0.73001
75	0.00188	-0.00093	1.99647	7.23562	0.72991	-77.12950	8.82242	-0.00031	0.37476	0.37108	0.73001
76	0.00188	0.00356	2.00083	-5.77963	0.72991	-90.71146	9.55949	0.00091	0.44142	0.43733	0.73001
77	0.00939	0.01323	1.99977	-9.40016	0.72991	-35.19206	5.98596	-0.00001	0.50833	0.50383	0.73001
78	0.02428	0.02790	1.99326	-9.68473	0.72991	-16.72704	4.16349	-0.00002	0.57414	0.56924	0.73001
79	0.04635	0.04732	1.98136	-9.54439	0.72991	-9.83290	3.22636	0.00004	0.63769	0.63240	0.73001
80	0.07527	0.07119	1.96424	-9.32605	0.72991	-6.55089	2.66397	0.00002	0.69918	0.69351	0.73001
81	0.11061	0.09911	1.94210	-9.09934	0.72991	-4.71248	2.28648	0.00010	0.75833	0.75227	0.73001
82	0.15184	0.13066	1.91526	-8.88147	0.72991	-3.56958	2.01371	0.00006	0.81472	0.80829	0.73001
83	0.19836	0.16534	1.88409	-8.67904	0.72991	-2.80011	1.80440	0.00005	0.86797	0.86117	0.73001
84	0.24948	0.20264	1.84905	-8.49323	0.72991	-2.25473	1.63745	0.00007	0.91771	0.91054	0.73001
85	0.30445	0.24199	1.81066	-8.32409	0.72991	-1.85132	1.49973	0.00002	0.96362	0.95610	0.73001
86	0.36244	0.28281	1.76947	-8.17229	0.72991	-1.54158	1.38266	0.00005	1.00547	0.99760	0.73001
87	0.42259	0.32451	1.72611	-8.03721	0.72991	-1.29898	1.28168	0.00002	1.04310	1.03490	0.73001
88	0.48400	0.36648	1.68126	-7.91869	0.72991	-1.10335	1.19272	0.00003	1.07642	1.06791	0.73001
89	0.54577	0.40811	1.63559	-7.81716	0.72991	-0.94192	1.11318	0.00003	1.10544	1.09666	0.73001
90	0.60694	0.44881	1.58985	-7.73333	0.72991	-0.80505	1.04104	0.00002	1.13023	1.12122	0.73001
91	0.66661	0.48802	1.54475	-7.66754	0.72991	-0.68626	0.97496	-0.00001	1.15094	1.14174	0.73001
92	0.72385	0.52518	1.50103	-7.62033	0.72991	-0.57872	0.91299	0.00002	1.16774	1.15842	0.73001
93	0.77778	0.55977	1.45944	-7.59221	0.72991	-0.47855	0.85406	0.00005	1.18087	1.17150	0.73001
94	0.82756	0.59133	1.42070	-7.58401	0.72991	-0.38119	0.79583	0.00004	1.19058	1.18123	0.73001
95	0.87238	0.61941	1.38549	-7.59680	0.72991	-0.28480	0.73677	-0.00004	1.19716	1.18789	0.73001
96	0.91150	0.64364	1.35449	-7.63029	0.72991	-0.18888	0.67522	0.00005	1.20095	1.19178	0.73001
97	0.94421	0.66367	1.32834	-7.68510	0.72991	-0.09377	0.60900	0.00005	1.20236	1.19333	0.73001
98	0.96987	0.67921	1.30765	-7.76932	0.72991	0.00102	0.53402	-0.00027	1.20196	1.19303	0.73001
99	0.98786	0.68998	1.29303	-7.90602	0.72991	0.09294	0.44683	0.00000	1.20046	1.19162	0.73001
100	0.99743	0.69565	1.28520	-8.16431	0.72991	0.18182	0.33717	0.00058	1.19894	1.19014	0.73001

```
PHI AT LEADING EDGE = -0.408047



AIRFOIL NO    1
   CD =  0.855350    CL= -3.195229    CM=  0.919994


AIRFOIL NO    2
   CD =  0.855699    CL=  3.195606    CM= -0.920369


TRAILING VORTICES DATA

  M    X1I(M)    Y1I(M)     X1(M)     Y1(M)     CIRC1     X2I(M)    Y2I(M)    X2(M)     Y2(M)     CIRC2

  1  1.422689 -0.515663  1.361533  0.660438  0.303002  1.422650  0.515679  1.361518  1.339601 -0.303004
  2  1.461441 -0.388789  1.297592  0.776672  0.188150  1.461422  0.388809  1.297593  1.223355 -0.188153
  3  1.355833 -0.340819  1.189576  0.734404  0.124051  1.355829  0.340817  1.189571  1.265596 -0.124049
  4  1.299884 -0.398826  1.192156  0.653853  0.091023  1.299864  0.398810  1.192130  1.346149 -0.091020
  5  1.284876 -0.463965  1.228392  0.597681  0.071417  1.284851  0.463949  1.228363  1.402325 -0.071417
  6  1.296405 -0.527009  1.281627  0.561994  0.058907  1.296365  0.526993  1.281587  1.438023 -0.058903
  7  1.339418 -0.584185  1.352604  0.552970  0.051384  1.339367  0.584180  1.352564  1.447062 -0.051380
  8  1.472104 -0.611886  1.464967  0.628781  0.047029  1.472065  0.611907  1.464953  1.371261 -0.047022
  9  1.549367 -0.535213  1.463863  0.737625  0.044248  1.549331  0.535249  1.463864  1.262425 -0.044245
 10  1.542382 -0.461499  1.406147  0.784009  0.042095  1.542347  0.461545  1.406156  1.216048 -0.042090
 11  1.540661 -0.391055  1.354445  0.831886  0.040097  1.540626  0.391112  1.354462  1.168180 -0.040094
 12  1.494399 -0.299028  1.256219  0.862878  0.038090  1.494463  0.299091  1.256309  1.137119 -0.038101
 13  1.428186 -0.241091  1.168522  0.855802  0.036077  1.428201  0.241126  1.168557  1.144212 -0.036063
 14  1.371032 -0.200555  1.099615  0.843087  0.034104  1.370976  0.200584  1.099597  1.156972 -0.034120
 15  1.321768 -0.169205  1.042795  0.829625  0.032317  1.321857  0.169211  1.042862  1.170314 -0.032310
 16  1.279626 -0.143227  0.994792  0.817524  0.030739  1.279589  0.143255  0.994785  1.182521 -0.030728
 17  1.243149 -0.120586  0.953129  0.807158  0.029352  1.243134  0.120613  0.953139  1.192870 -0.029345
 18  1.210979 -0.100274  0.916140  0.798257  0.028095  1.210928  0.100306  0.916127  1.201802 -0.028102
 19  1.181323 -0.081971  0.882343  0.789756  0.026895  1.181348  0.081979  0.882366  1.210232 -0.026899
 20  1.153538 -0.065449  0.851128  0.781354  0.025751  1.153549  0.065448  0.851134  1.218636 -0.025770
 21  1.127092 -0.050627  0.822065  0.772729  0.024655  1.127158  0.050604  0.822094  1.227207 -0.024633
 22  1.101864 -0.037430  0.795015  0.763843  0.023571  1.101785  0.037471  0.794989  1.236241 -0.023537
 23  1.077483 -0.025962  0.769796  0.754360  0.022493  1.077426  0.025998  0.769782  1.245707 -0.022504
 24  1.054038 -0.016268  0.746501  0.744310  0.021526  1.054016  0.016282  0.746496  1.255715 -0.021528
```

25	1.031467	-0.008006	0.724843	0.733888	0.020542	1.031455	0.008007	0.724835	1.266121	-0.020560
26	1.010293	-0.002605	0.706208	0.722474	0.019656	1.010291	0.002604	0.706206	1.277526	-0.019699

LIST OF REFERENCES

1. Teng, N.H., *The Development of a Computer Code (U2DIIF) for the Numerical Solution of Unsteady, Inviscid and Incompressible Flow over an Airfoil,* Master's Thesis, Naval Postgraduate School, Monterey, California, Jun 1987, pp. 1-135.

2. Krainer, A., *A Numerical Method For Computing Unsteady 2-D Boundary Layer Flows,* Naval Postgraduate School, Monterey, California, Feb 1988, pp. 5-11.

3. Hess, J.L. and Smith, A.M.O., *Calculation of Potential Flow about Arbitrary Bodies,* Progress in Aeronautical Sciences, Vol. 8, Pergamon Press, Oxford, 1966, pp. 1-138.

4. Basu, B.C. and Hancock, G.J., *The Unsteady Motion of a Two-Dimensional Aerofoil in Incompressible Inviscid Flow,* Journal of Fluid Mechanics, Vol. 87, Jul 1978, pp. 159-168.

5. Giesing, J.P., *Unsteady Two-Dimensional Potential Flow for Two Bodies with Lift,* Technical Report Contract No. 4308(00), Report No. DAC-33552, Douglas Aircraft Division, Long Beach, California, Mar 1967, pp. 1-60.

6. Krainer, A., Naval Postgraduate School, Monterey, California. Private Communication, Mar-Sept 1988.

7. Giesing, J.P., *Unsteady Two-Dimensional Potential Flow with Lift,* Technical Report, Report No. LB-32144, Douglas Aircraft Division, Long Beach, California, Mar 1965, pp. 1-67.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, VA 22304-6145	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, CA 93943-5002	2
3.	Chairman, Dept. of Aeronautics, Code 67 Naval Postgraduate School Monterey, CA 93943-5000	1
4.	Professor M.F. Platzer Naval Postgraduate School Monterey, CA 93943-5000	10
5.	Professor R.M. Howard Naval Postgraduate School Monterey, CA 93943-5000	1
6.	Mr. Andreas Krainer Viktor Adlerstr.52 A-2345 Brunn. Austria	1
7	Mr. Pang Chung Khiang Air System Program Office Defence Material Organisation Paya Lebar Airport Singapore 1953 Republic Of Singapore	5
8	Professor T. Cebeci Staff Director Research & Technology McDonnell Douglas Corporation 3855 Lakewood Boulevard Long Beach, CA 90846	1
9	Mr. Joseph Giesing Senior Staff Engineer Airframe Technology McDonnell Douglas Corporation 3855 Lakewood Boulevard Long Beach, CA 90846	1

10 Mr. James O'Brien 1
Research Scientist
Heat Transfer Branch
Mail Stop 5-11
NASA-Lewis Research Center
21000 Brookpark Road
Cleveland, OH 44135

11 Mr. Campbell Henderson 1
Head, Aircraft Performance Analysis Branch
Code 6051, Naval Air Development Center
Warminster, PA 18974-5000

12 Ms. Lisa J. Cowles 1
Code 6051, Naval Air Development Center
Warminster, PA 18974-5000

13 Mr. H.A. Berman 1
Code 931M, Naval Air System Command
Washington, DC 20361

14 Mr. G. Derderian 1
Code 931E, Naval Air System Command
Washington, DC 20361

Thesis
P14571
c.1

Pang

A computer code (USPOTF2) for unsteady incompressible flow past two airfoils.

thesP14571
A computer code (USPOTF2) for unsteady i
3 2768 000 84412 0
DUDLEY KNOX LIBRARY